Quantum non-malleability and authentication

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Joint work with Gorjan Alagic, NIST and University of Maryland

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Motivation: a classical story...

Crypto for bank transfers

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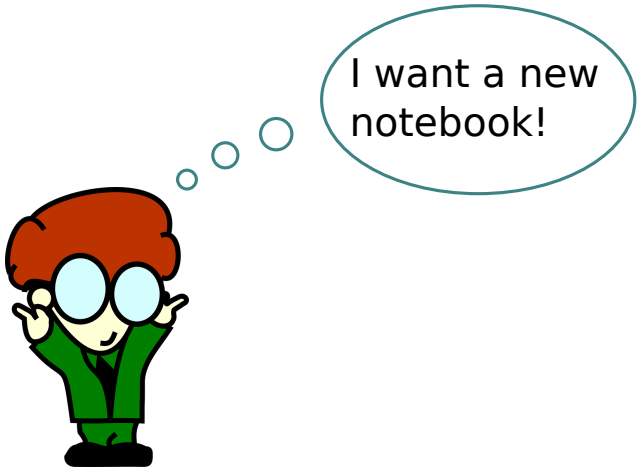

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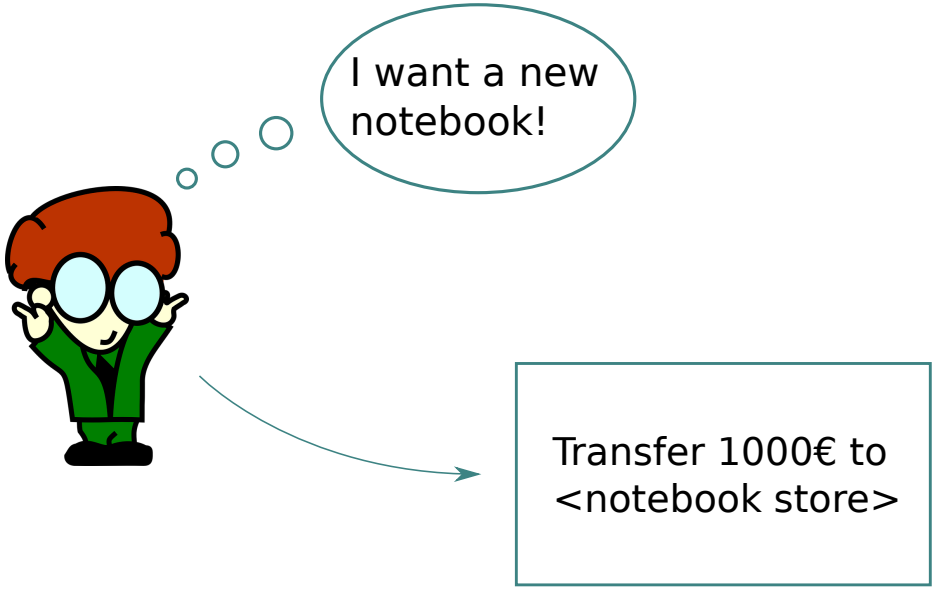

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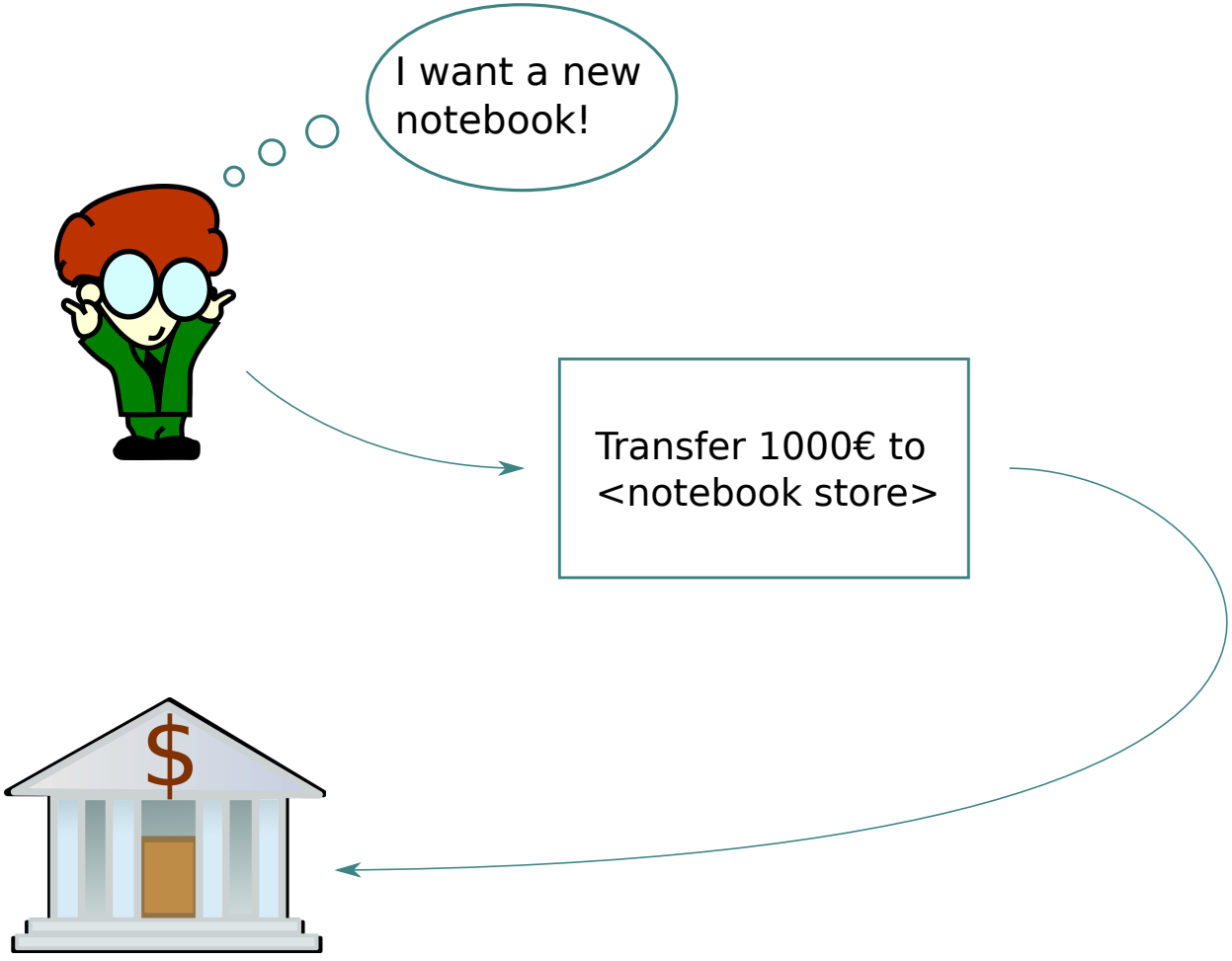

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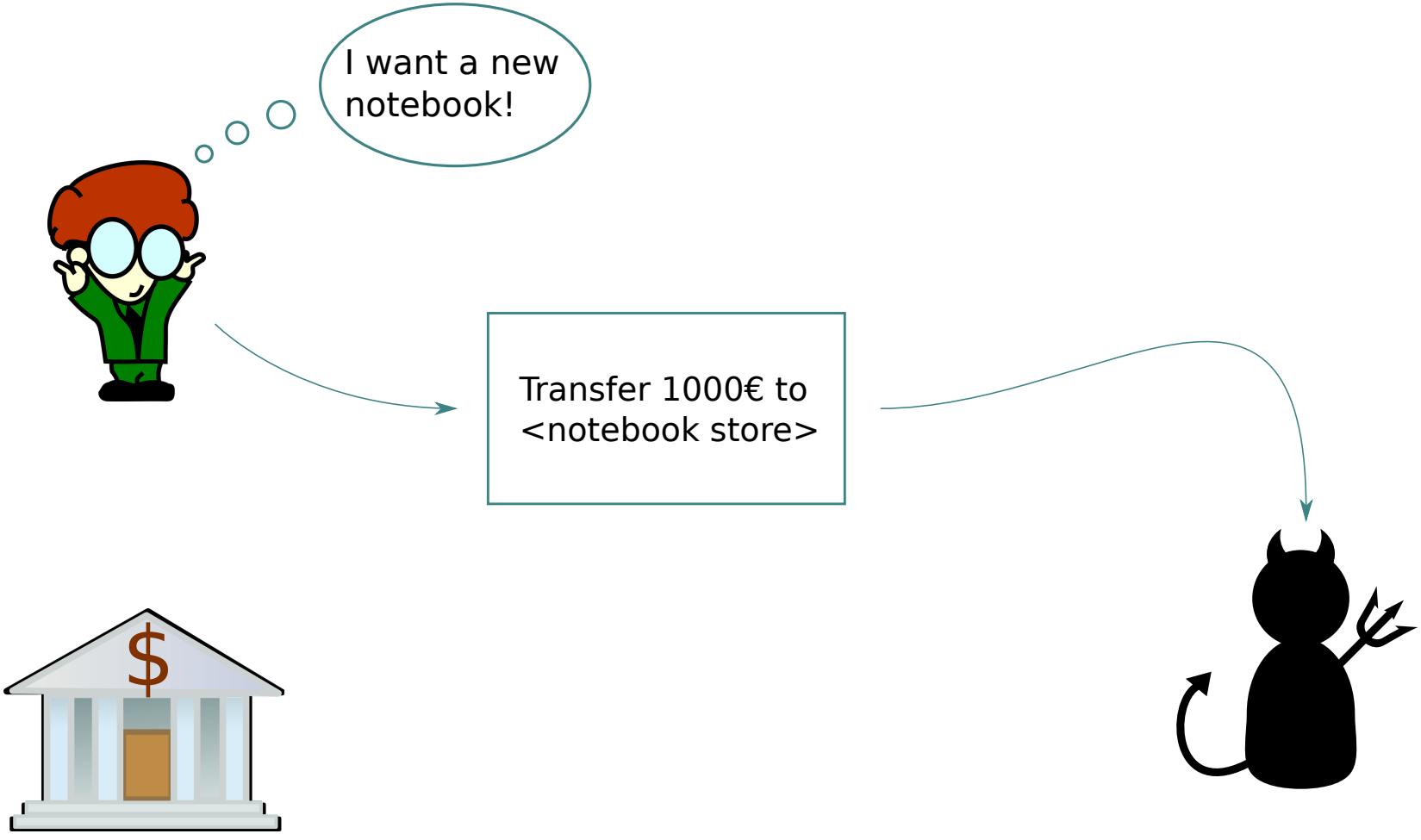

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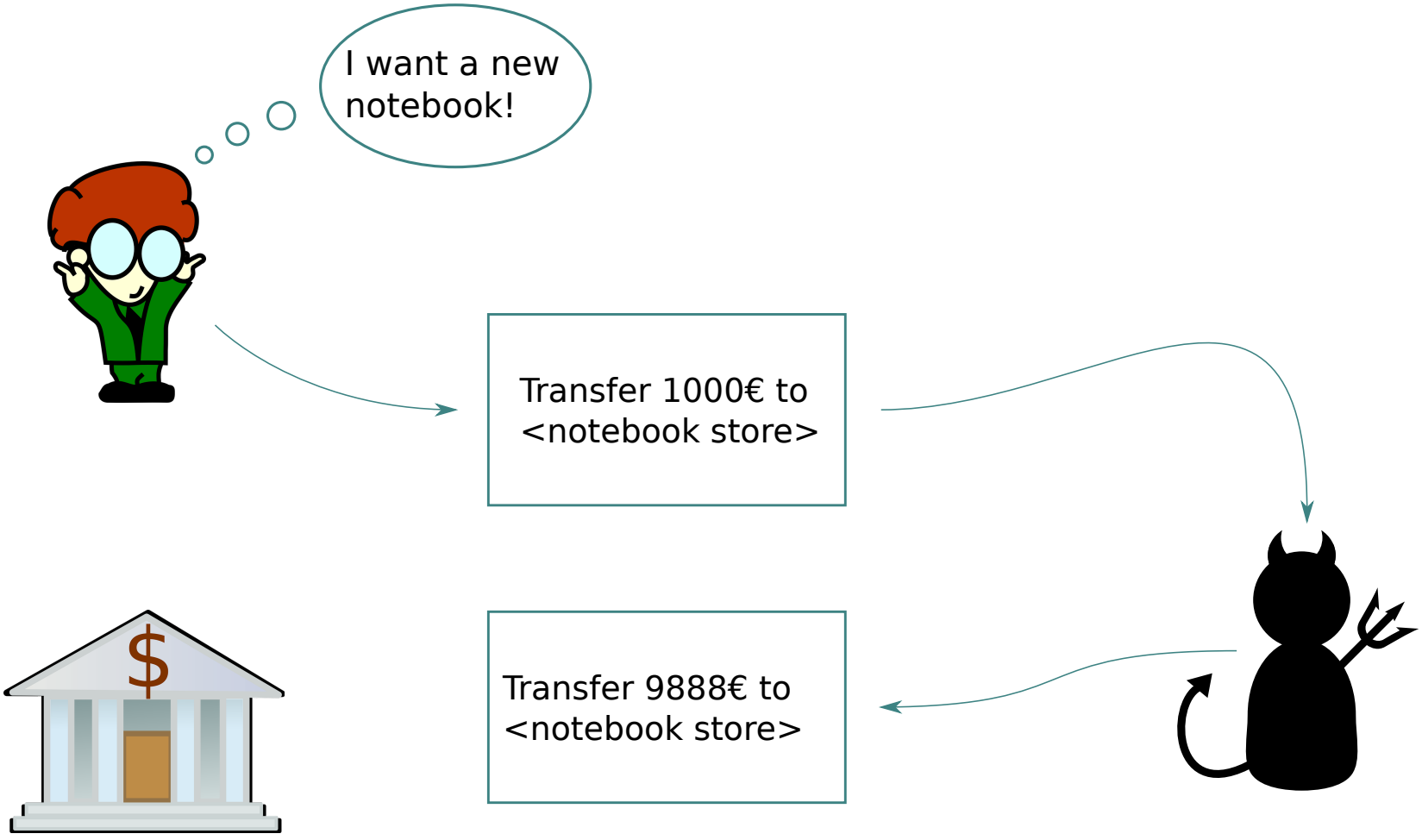

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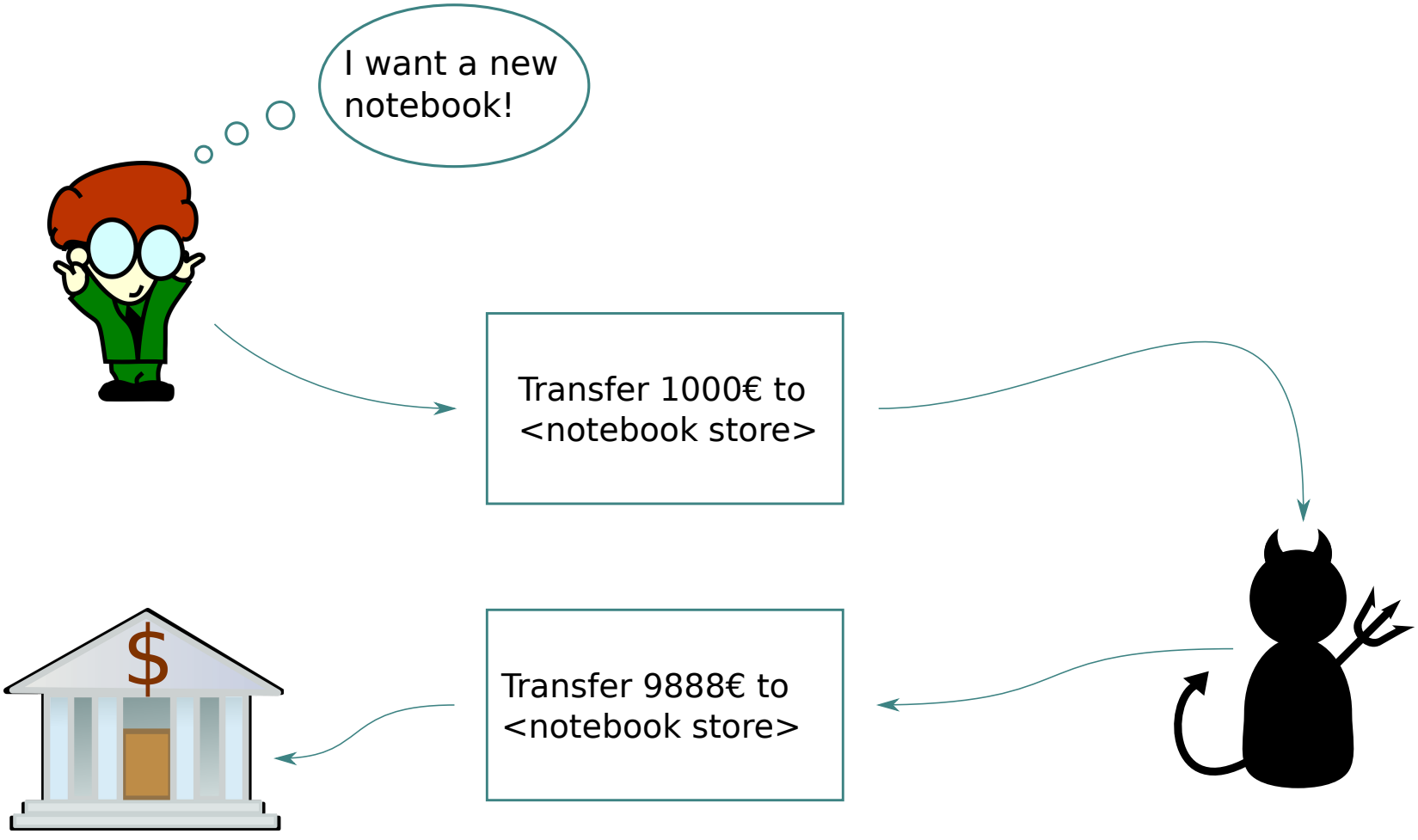

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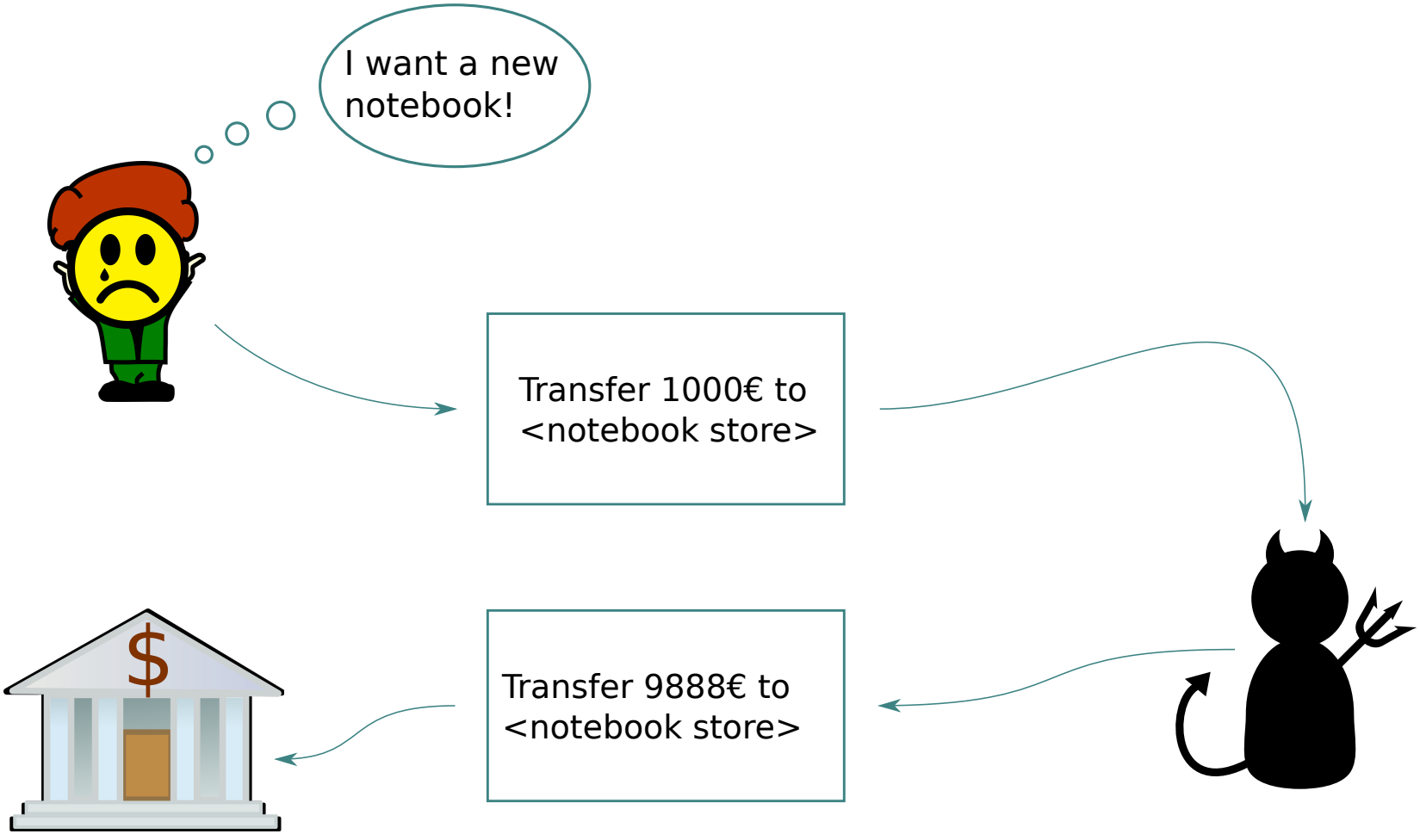

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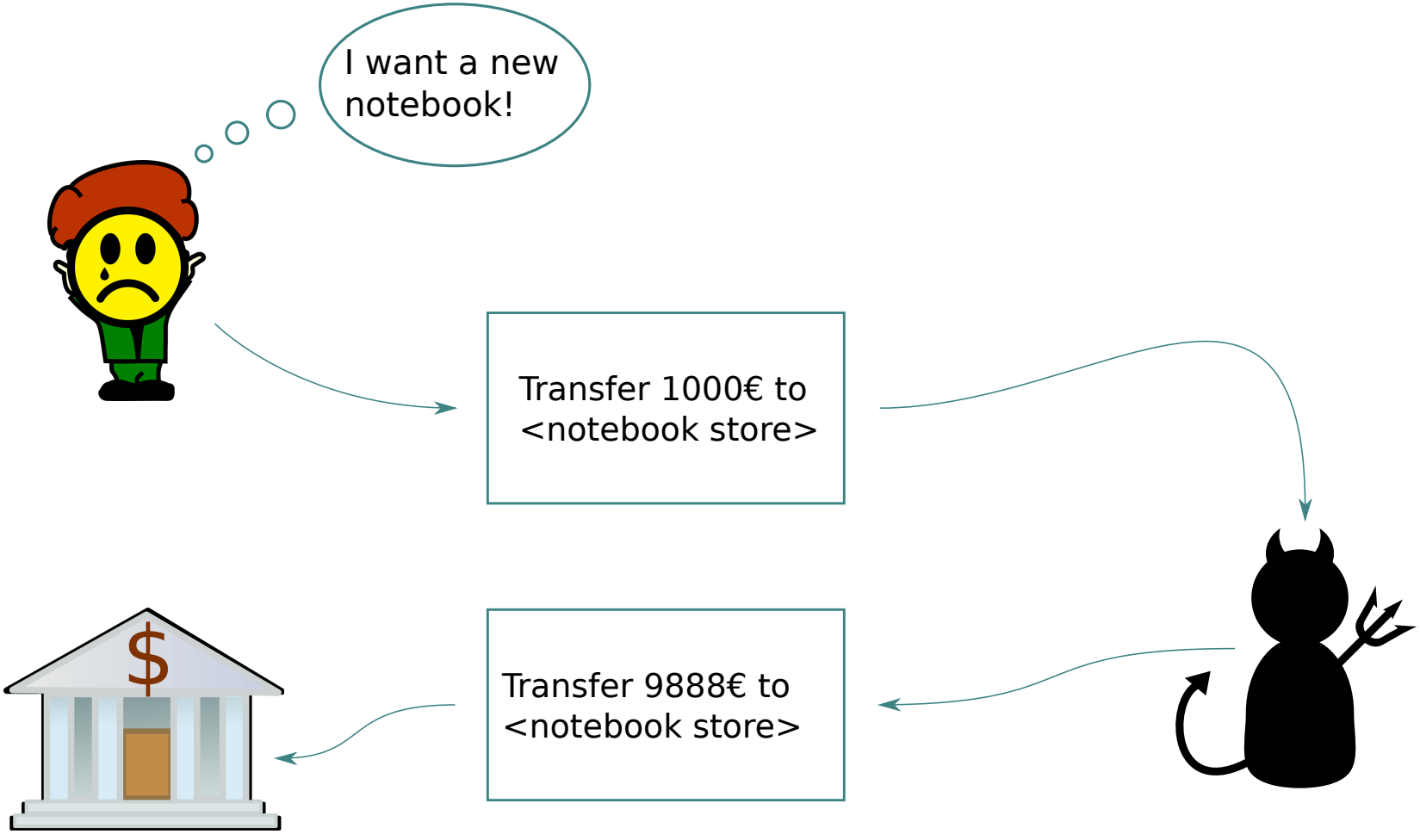

- What cryptographic security notions would fix this problem?

Non-malleability

- One solution is non-malleable encryption:

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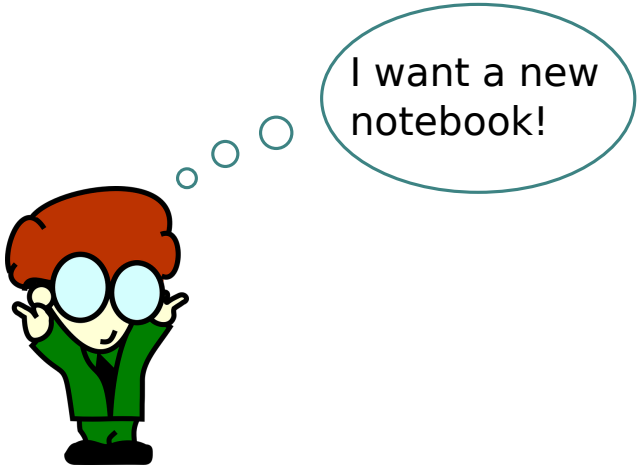

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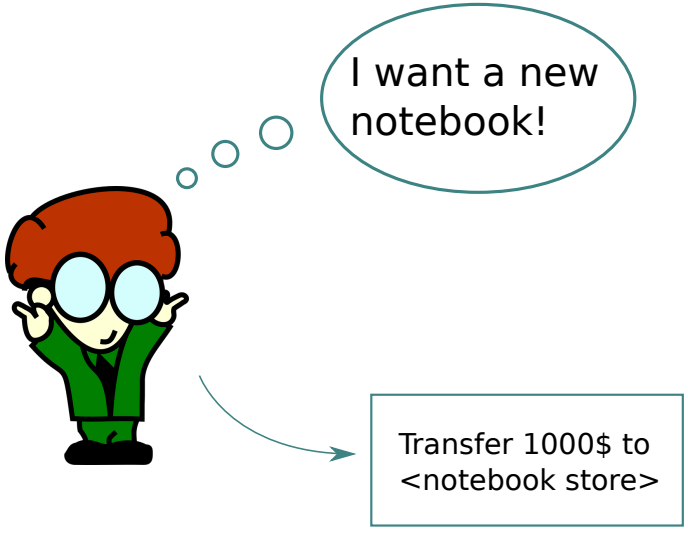

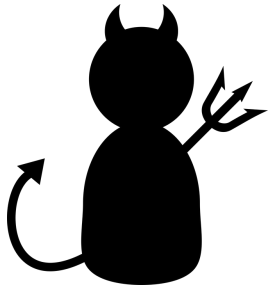

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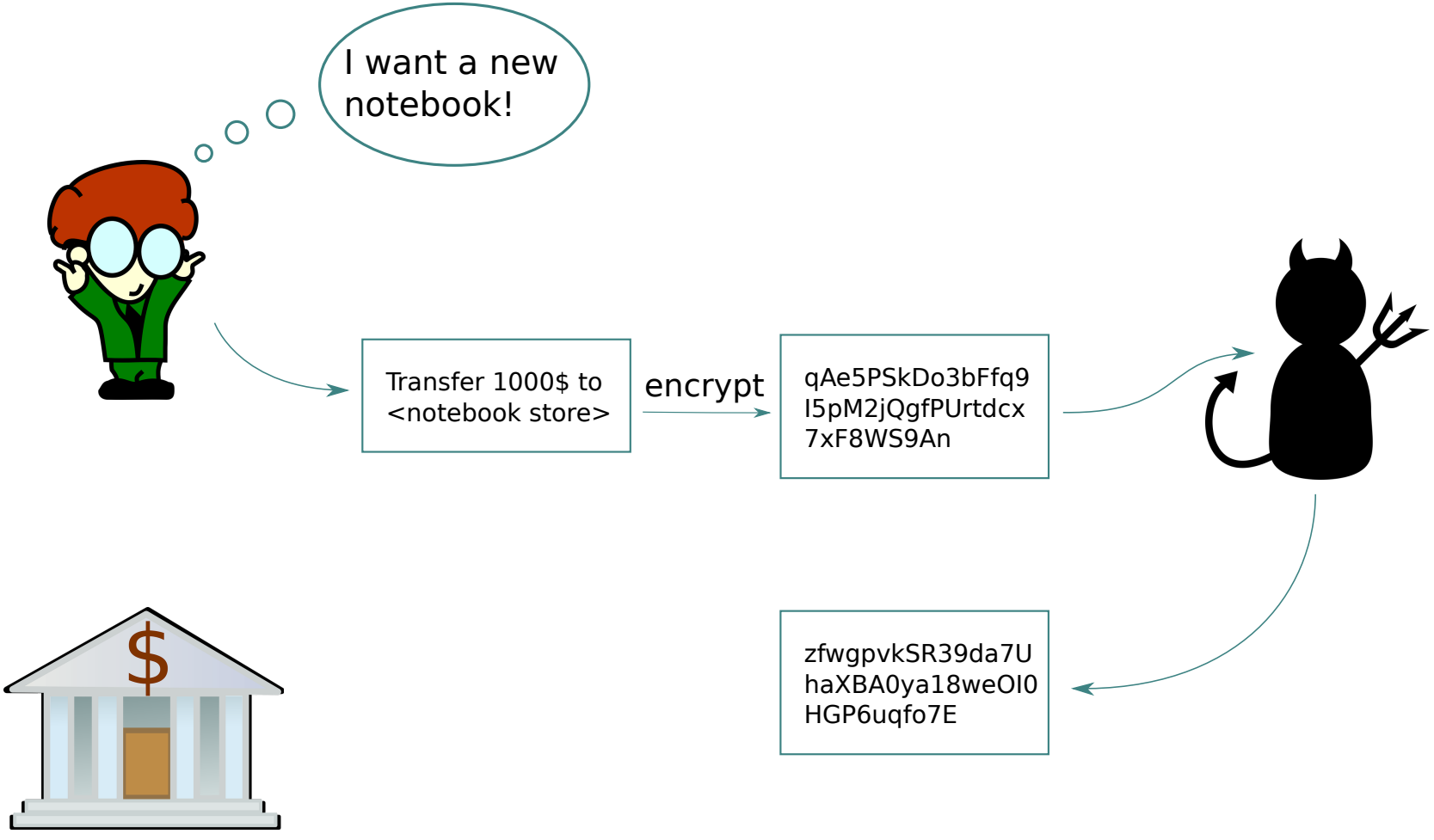

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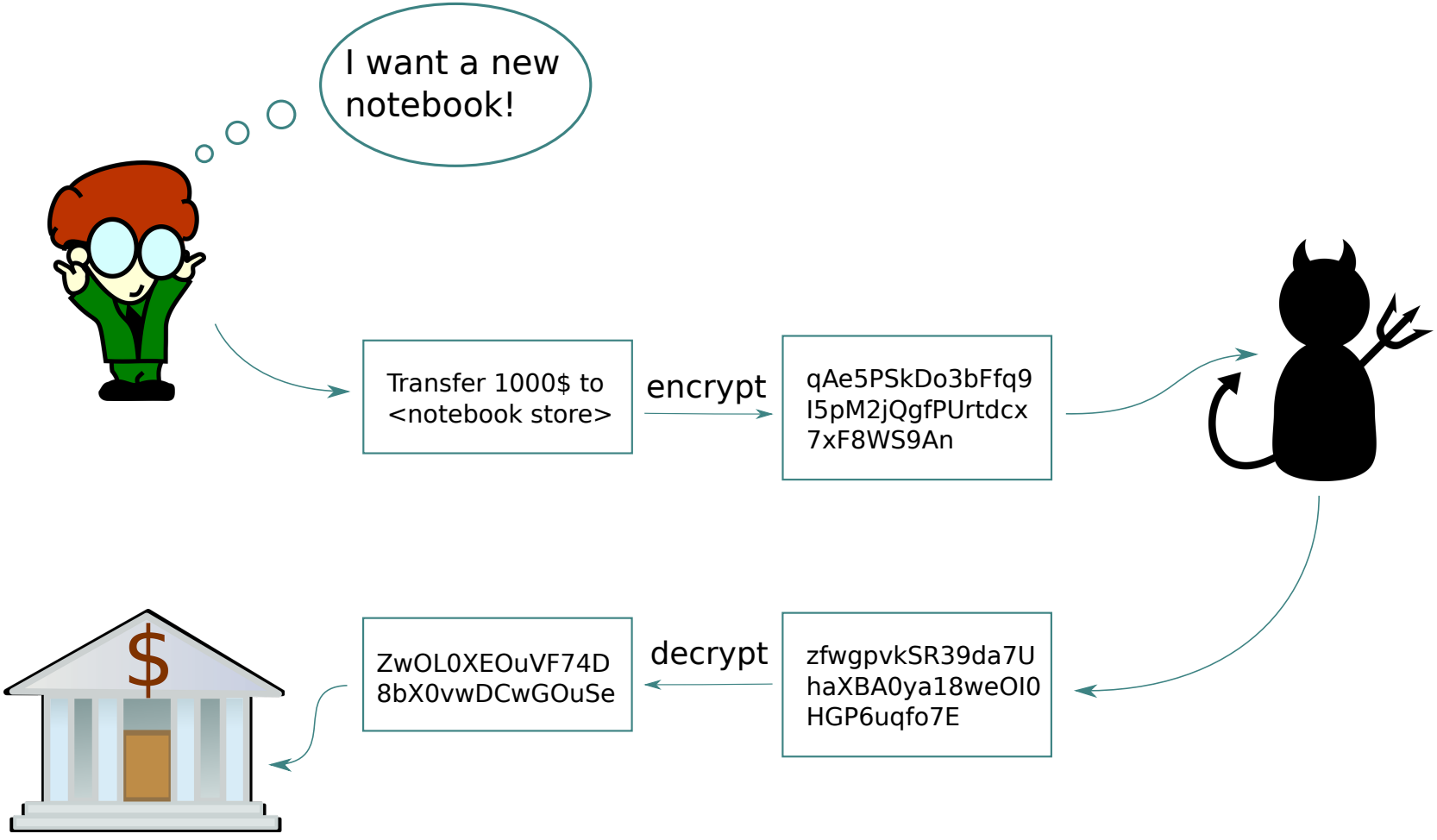

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New definition of information-theoretic quantum non-malleability which

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- implies secrecy, analogously to quantum authentication
- serves as a primitive for building quantum authentication
- has both a simulation-based and an entropic characterization
- ♠ Additional result: The new definition of quantum authentication with key recycling (Garg, Yuen, Zhandry '16, next talk!) can be fulfilled using unitary 2-designs.

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  (X, C), $(\tilde{X}, \tilde{C})$ two plaintext ciphertext pairs, $C \neq \tilde{C}$

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- later $\approx$simulation-based definition (McAven, Safavi-Naini, Yung '04)

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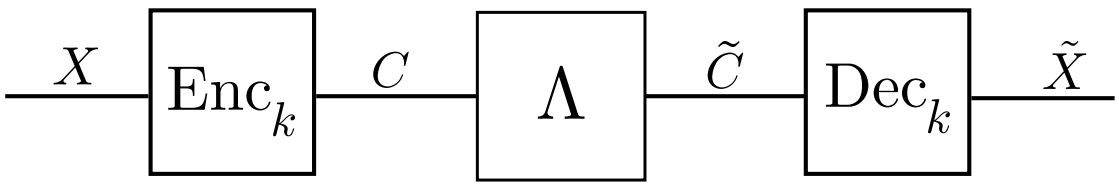

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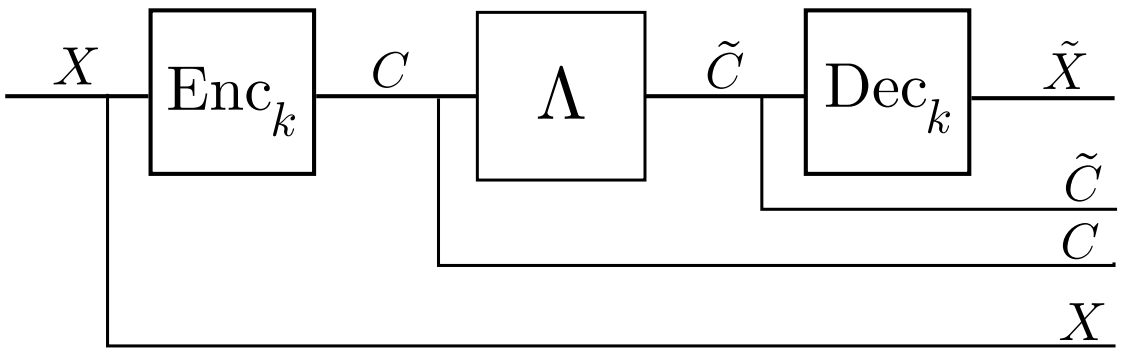

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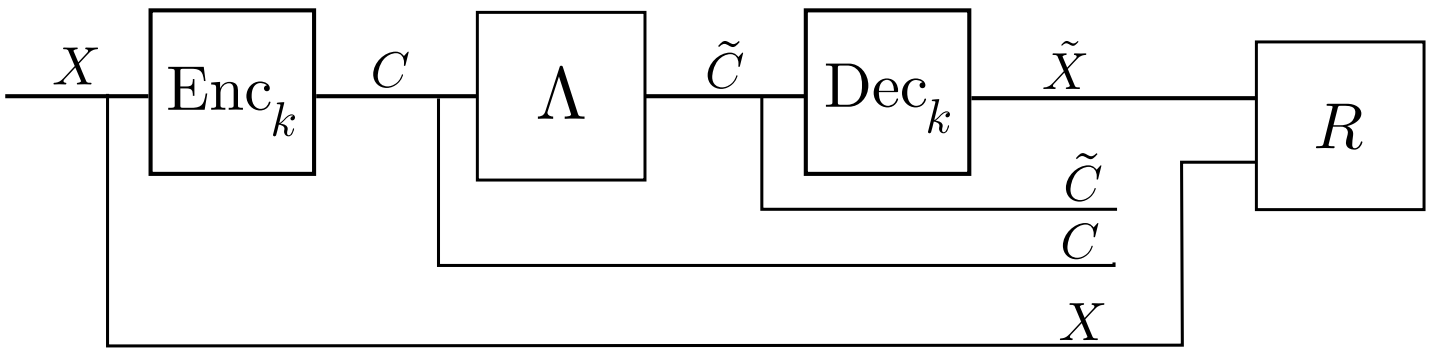

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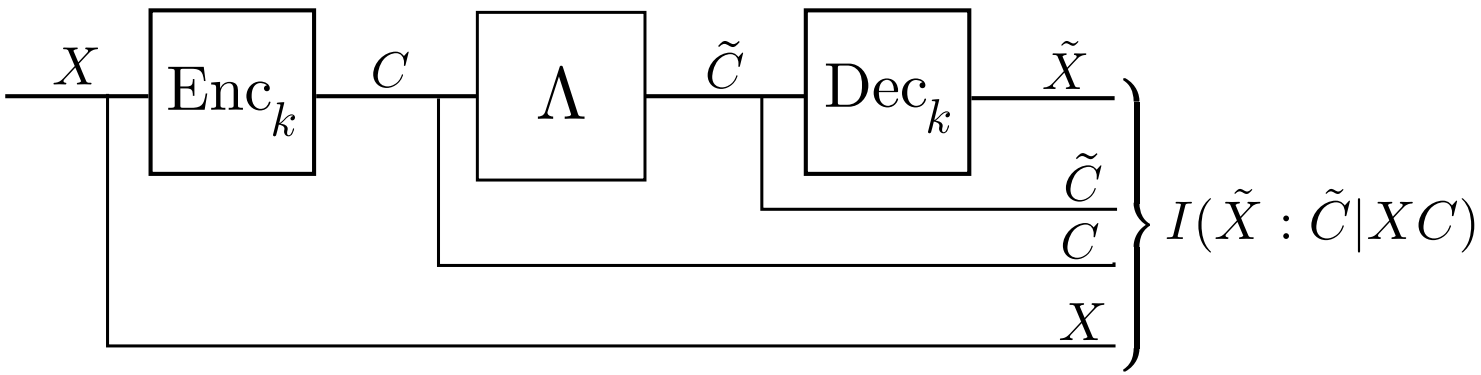

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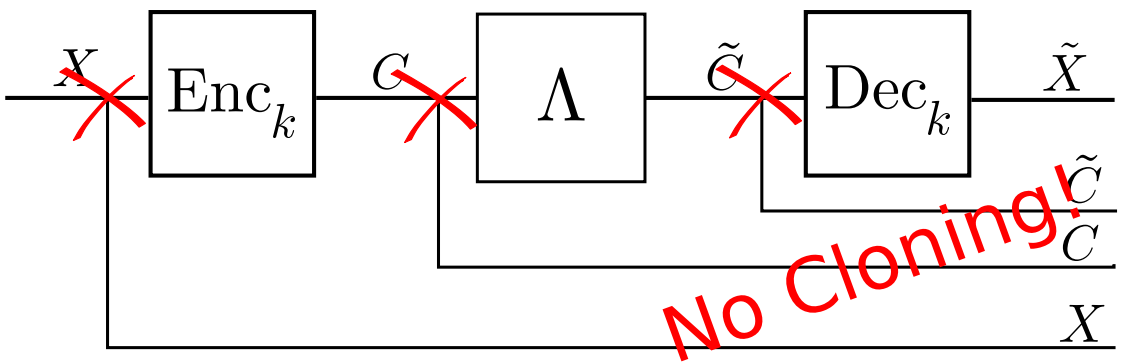

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def: Quantum encryption scheme: $(\mathrm{Enc}_k, \mathrm{Dec}_k)$

- classical uniformly random key k
- encryption map $(\mathrm{Enc}_k)_{A\to C}$, decryption map $(\mathrm{Dec}_k)_{C\to \bar{A}}$

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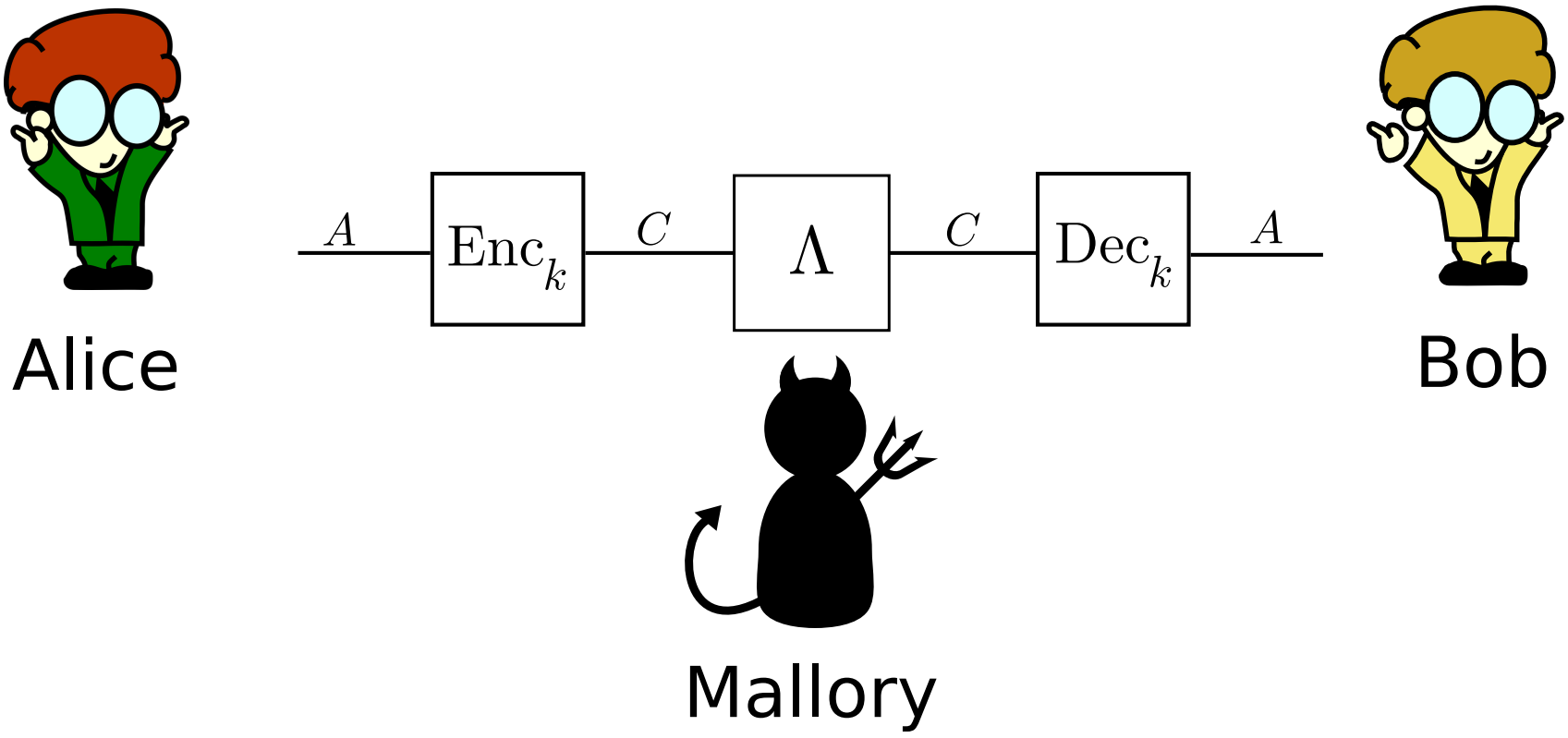
- correctness: $\mathrm{Dec}_k \circ \mathrm{Enc}_k = \mathrm{id}_A$
- average encryption map: $\mathrm{Enc}_K = \mathbb{E}_k \mathrm{Enc}_k$

Setup for q-non-malleability

- Recall: classical non-malleability setup



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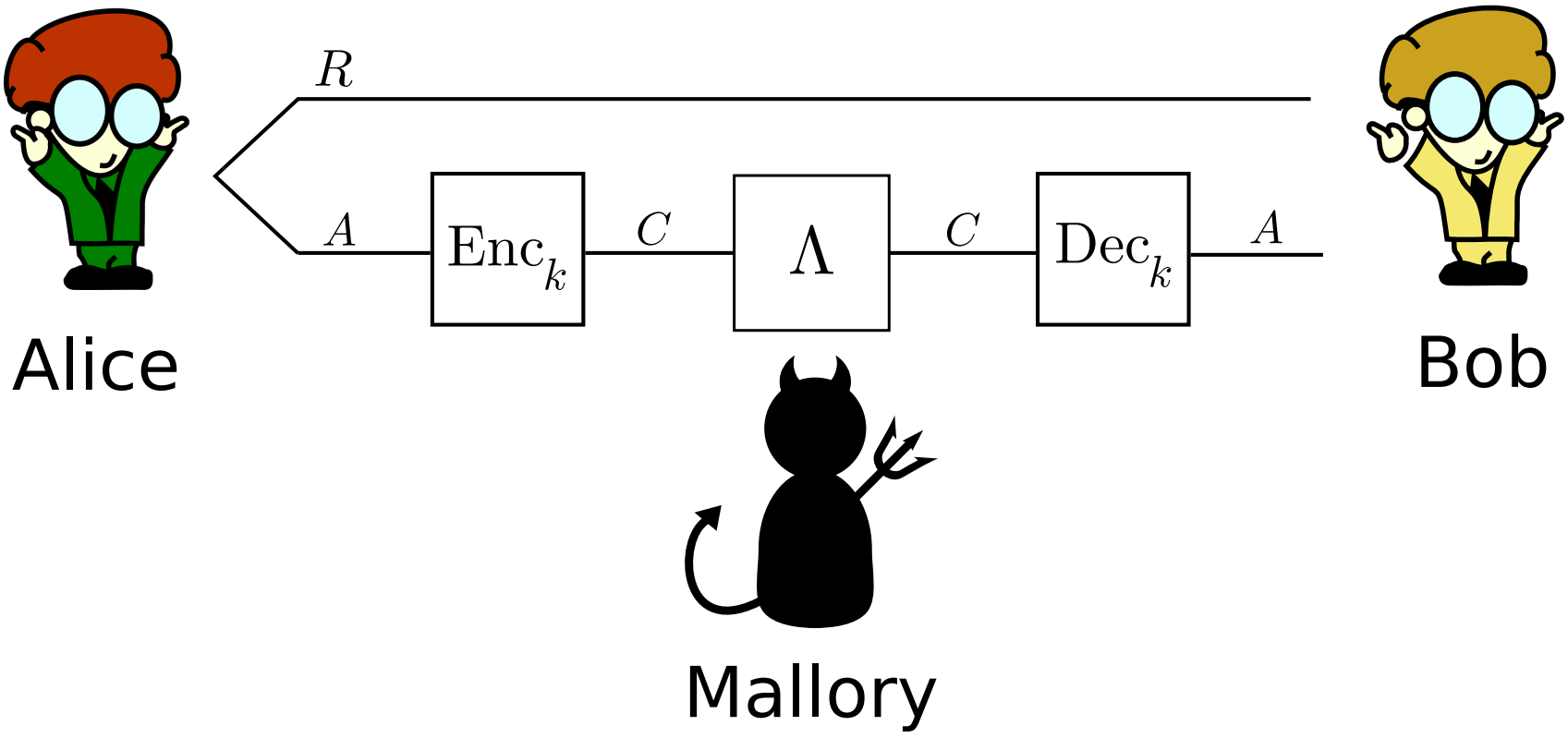

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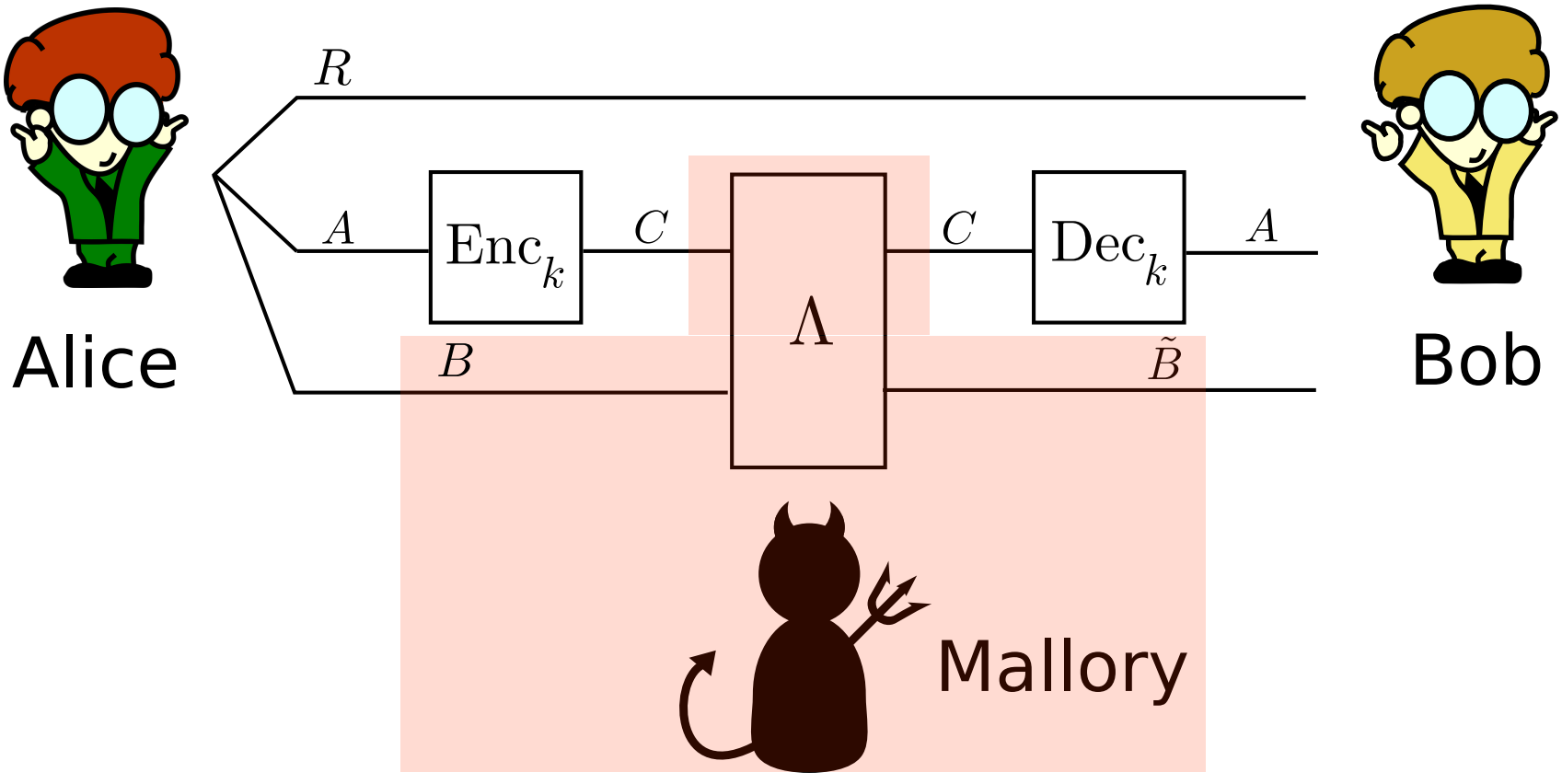

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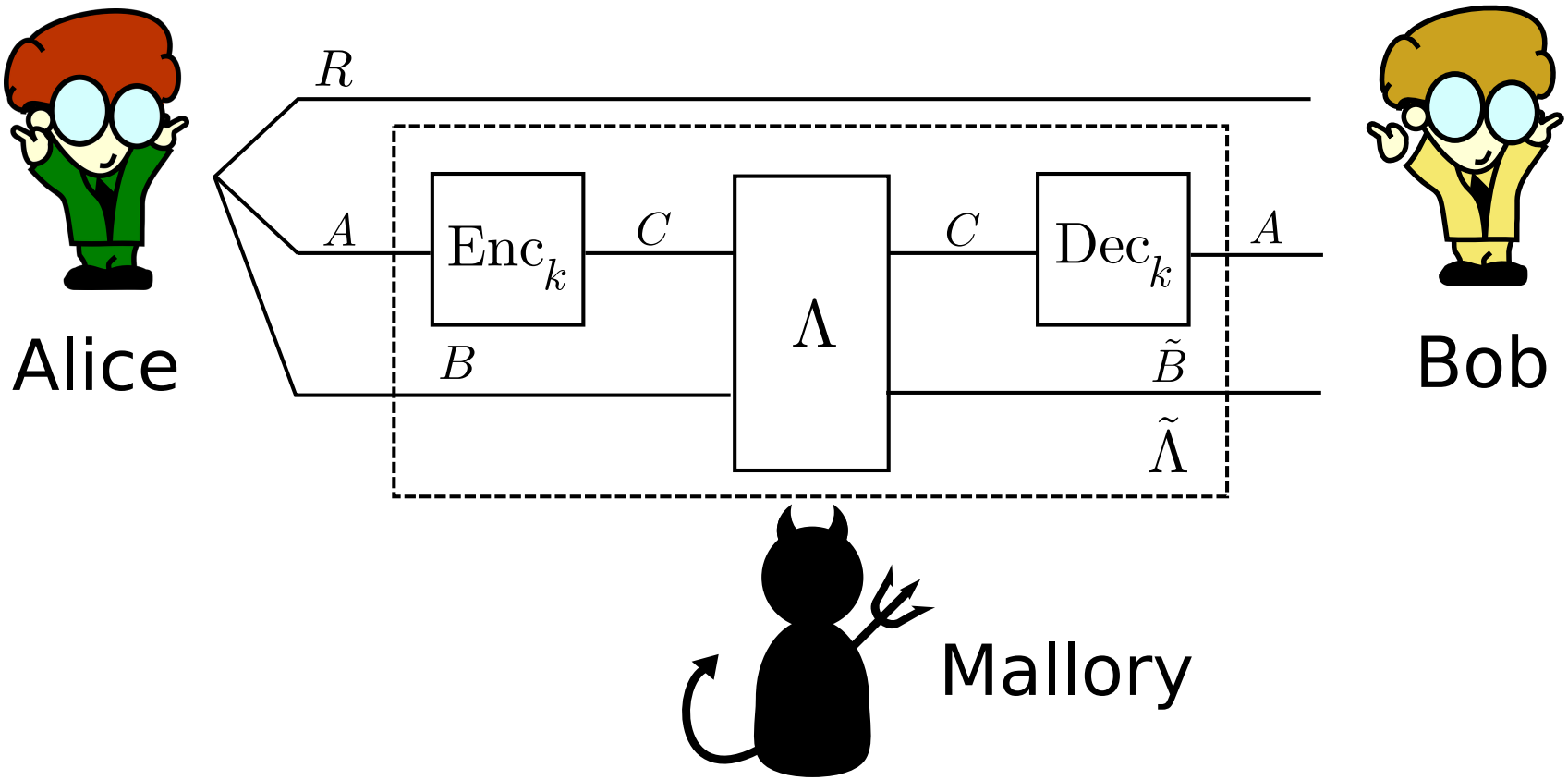
def: effective map on plaintexts and side info

$\tilde{\Lambda} = \mathbb{E}_k[\mathrm{Dec}_k \circ \Lambda \circ \mathrm{Enc}_k]$



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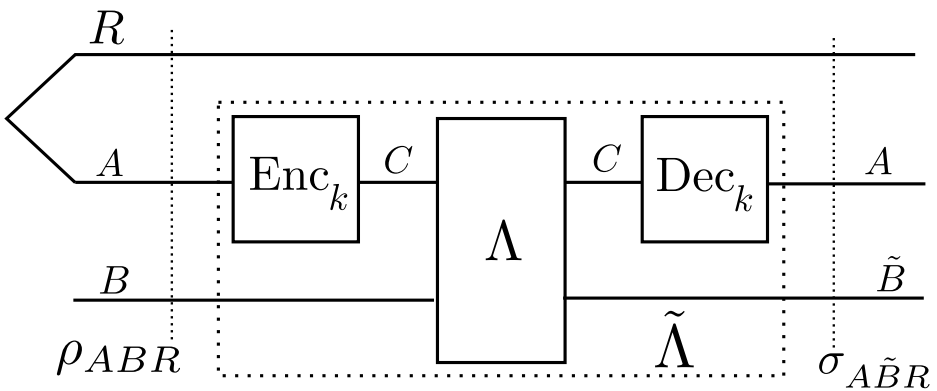
Definition (Quantum non-malleability (qNM))

A scheme $\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *is non-malleable, if for all states* ρ_{ABR} *and all attacks* $\Lambda_{CB\to C\tilde{B}}$,

$$I(AR:\tilde{B})_\sigma \leq I(AR:B)_\rho$$

with $\sigma_{A\tilde{B}R} = \tilde{\Lambda}_{AB\to A\tilde{B}}(\rho_{ABR})$.



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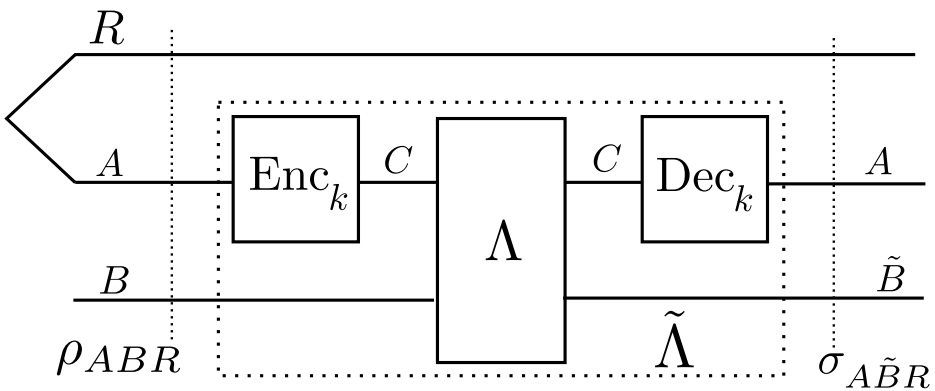

$$p_=(\Lambda,\rho) = F(\mathrm{tr}_{\tilde{B}}\Lambda_{CB\to C\tilde{B}}(|\phi^+\rangle\langle\phi^+|_{CC'}\otimes\rho_B), |\phi^+\rangle\langle\phi^+|_{CC'})^2$$

Alternative characterization

- qNM can be characterized in the simulation picture!

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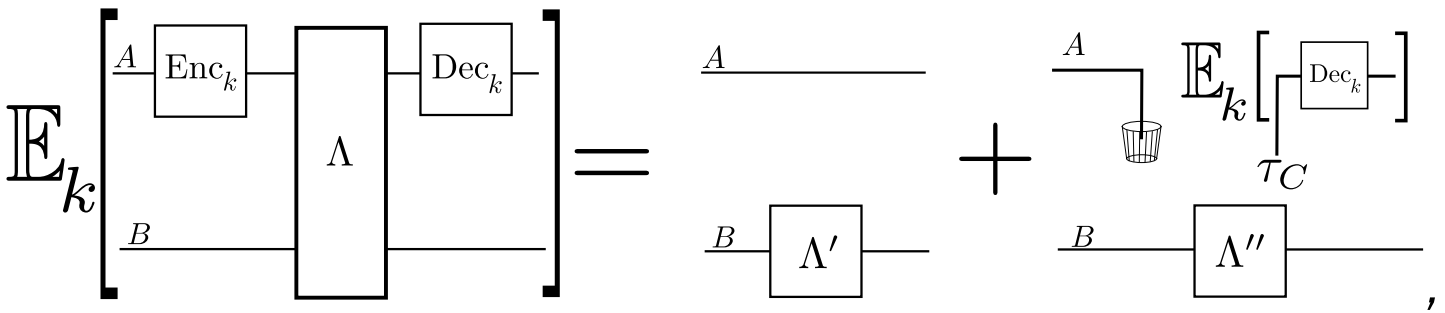
Theorem (Alagic, CM)

Let $\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *be a quantum encryption scheme.* Π *is qNM if and only if*



where Λ' *and* Λ'' *are explicitly given in terms of* Λ*.*

Earlier definition

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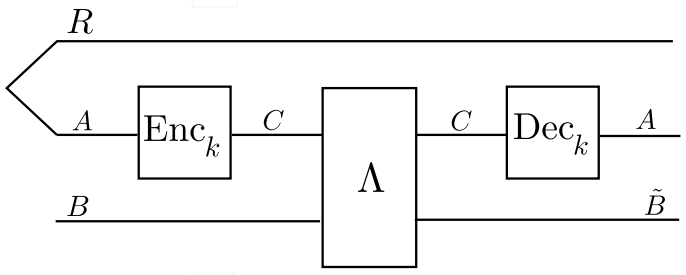
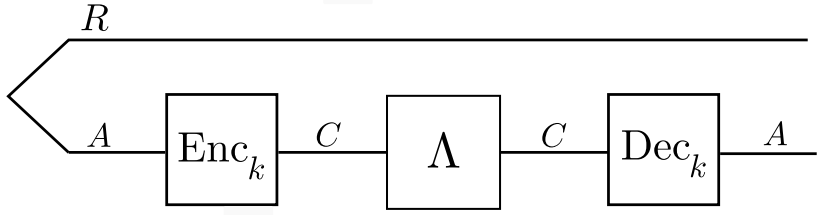


Ambainis, Bouda and Winter '09



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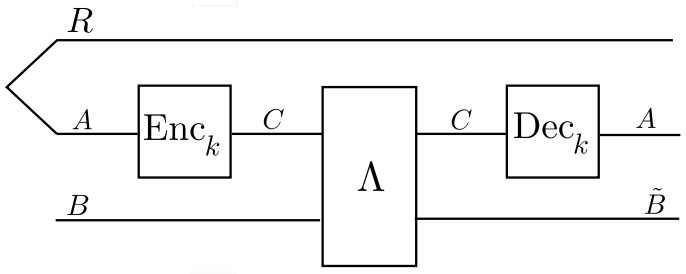

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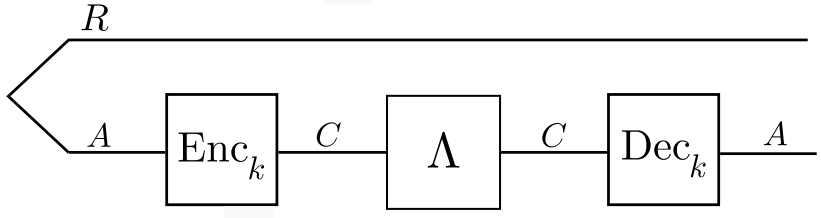

Simulator:

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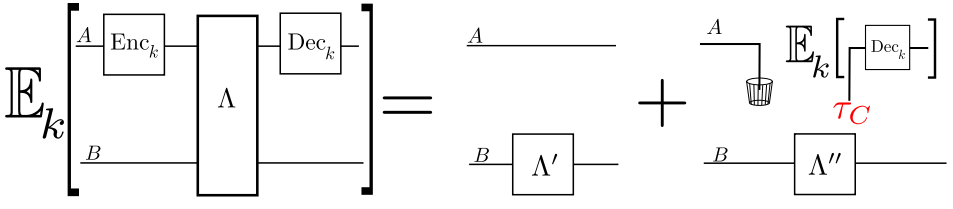

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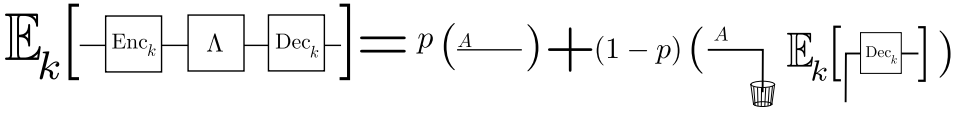

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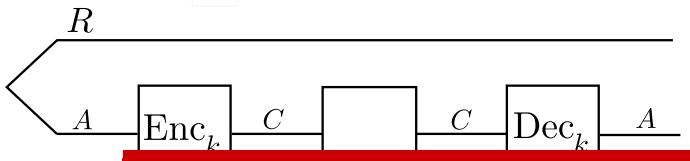

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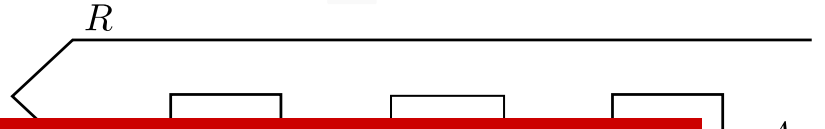

Simul

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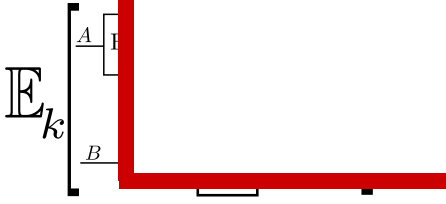

Separating scheme: ABW-NM allows "plaintext injection" attack, qNM prevents it

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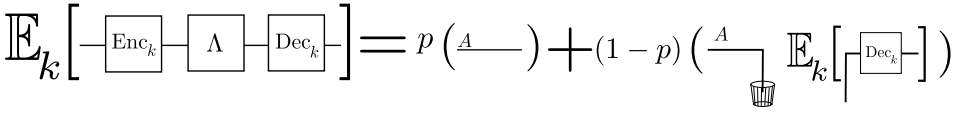

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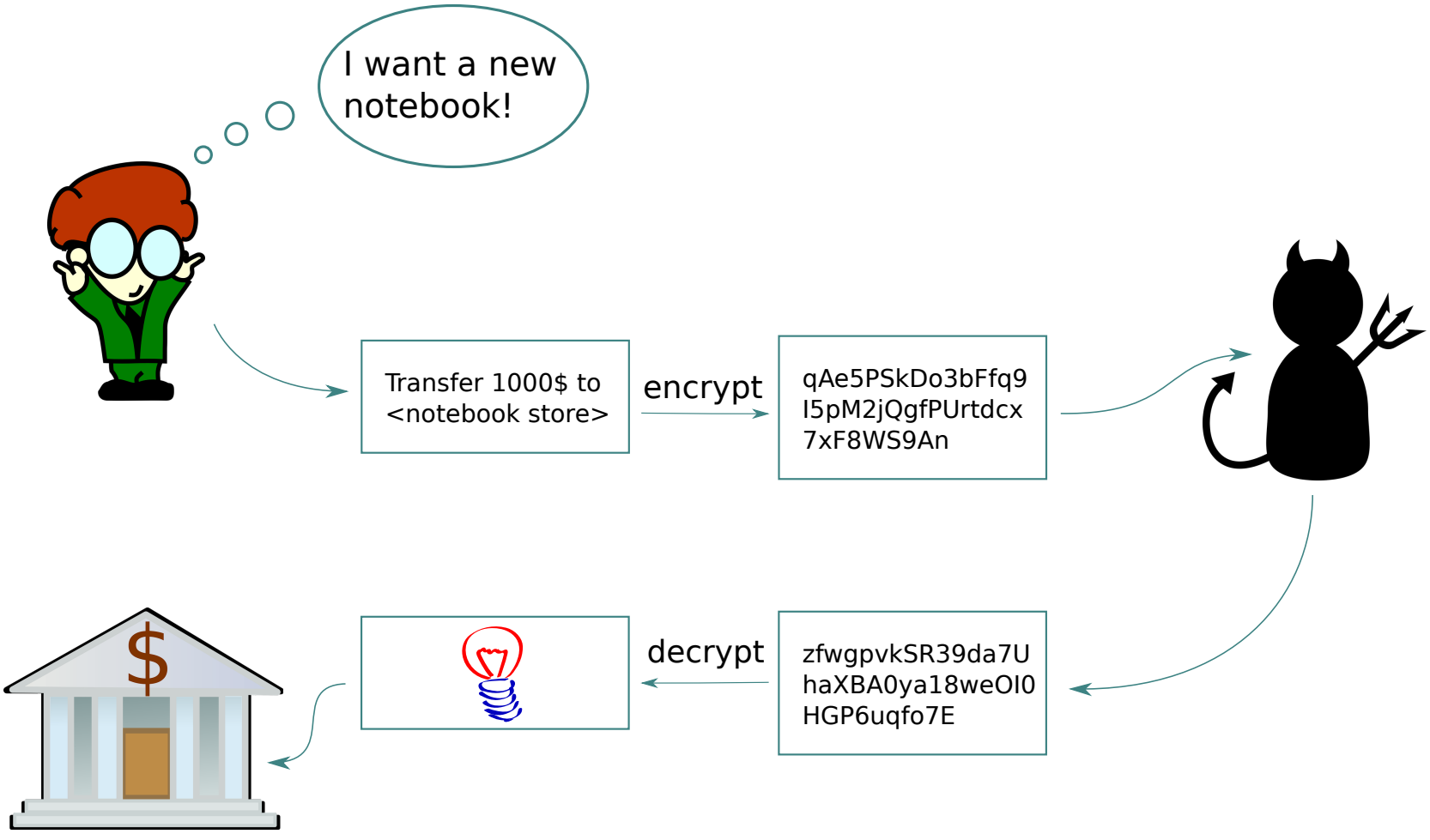
  qNM$\Leftrightarrow$ $\{\mathrm{Enc}_k\}_k$ is *unitary 2-design*($\Leftrightarrow$ ABW-NM, Ambainis et al.)
- non-unitary schemes are interesting, e.g. for authentication.
- ! qNM $\Rightarrow$ information theoretic IND
- qNM serves as primitive for quantum authentication schemes $\Rightarrow$ last part of the talk

Summary non-malleability

	ABW-NM	qNM
assumes secrecy	✓	✗
implies secrecy	✗	✓
secure against plaintext injection	✗	✓
primitive for authentication	✗	✓

Authentication

Authentication



Quantum authentication

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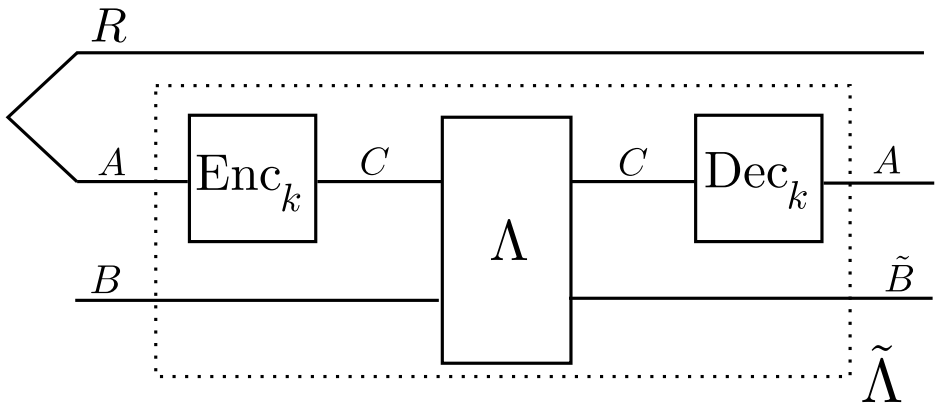
Definition (GYZ Authentication; Garg, Yuen and Zhandry)

$\Pi = (\mathrm{Enc}_k, \mathrm{Dec}_k)$ *is* ε*-GYZ-authenticating if, for any attack* $\Lambda_{CB\to CB'}$*, there exists* $\Lambda^{acc}_{B\to\tilde{B}}$ *such that for all* ρ_{AB}

$$\mathbb{E}_k \left[\left\| \Pi_{acc} \left[\mathrm{Dec}_k \circ \Lambda \circ \mathrm{Enc}_k(\rho_{AB}) \right] \Pi_{acc} - \left(\mathrm{id}_A \otimes \Lambda^{acc} \right)(\rho_{AB}) \right\|_1 \right] \leq \varepsilon$$

with $\Pi_{acc} = \mathbb{1} - \perp$.



GYZ-authentication with 2-designs

- GYZ authenticating scheme from 8-designs (GYZ '16)

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- advantages: shorter keys, nice constructions (Clifford group)

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consider pure states and attack isometries (Stinespring)

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want to bound

$$\mathbb{E}_{k}\left[\left\|\langle 0|_{T} U_{k}^{\dagger} V U_{k} \left(|\psi\rangle_{AB} \otimes |0\rangle_{T}\right) - \Gamma^{V} |\psi\rangle_{AB}\right\|_{2}^{2}\right]$$

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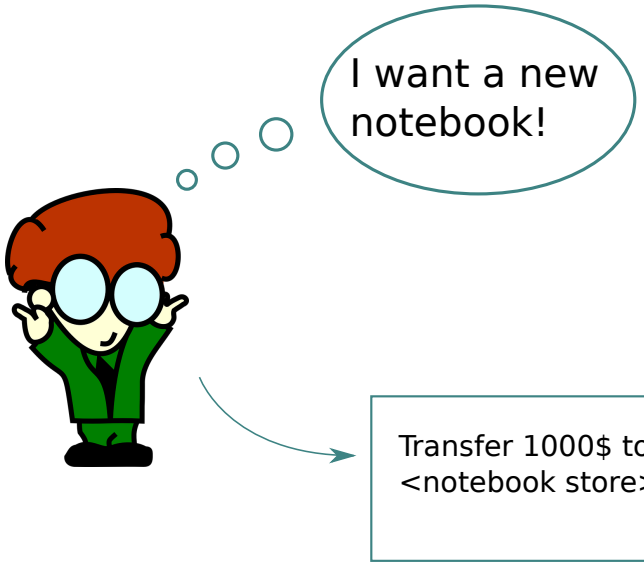
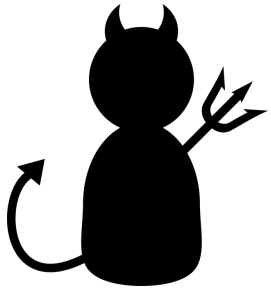
Use "swap trick" $\mathrm{tr} A_X B_X = \mathrm{tr} S_{XX'} A_X \otimes B_{X'}$ and Schur's lemma for $U \mapsto U \otimes U$

Authentication from NM: Intuition





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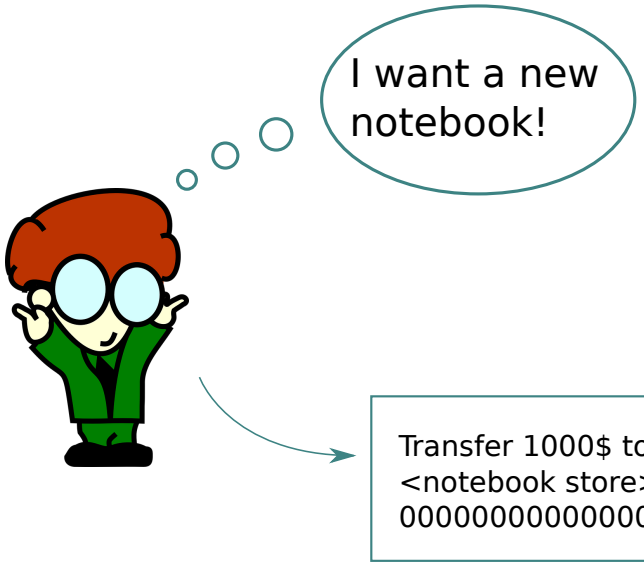
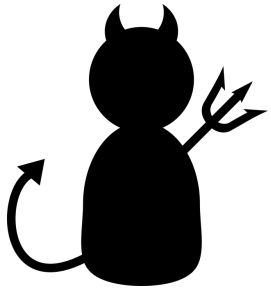
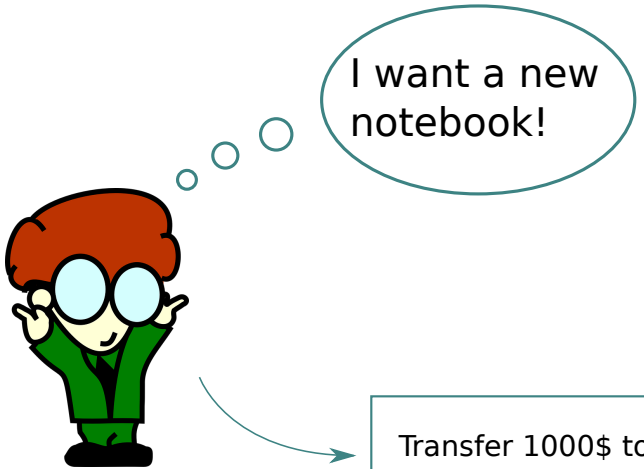




Authentication from NM: Intuition

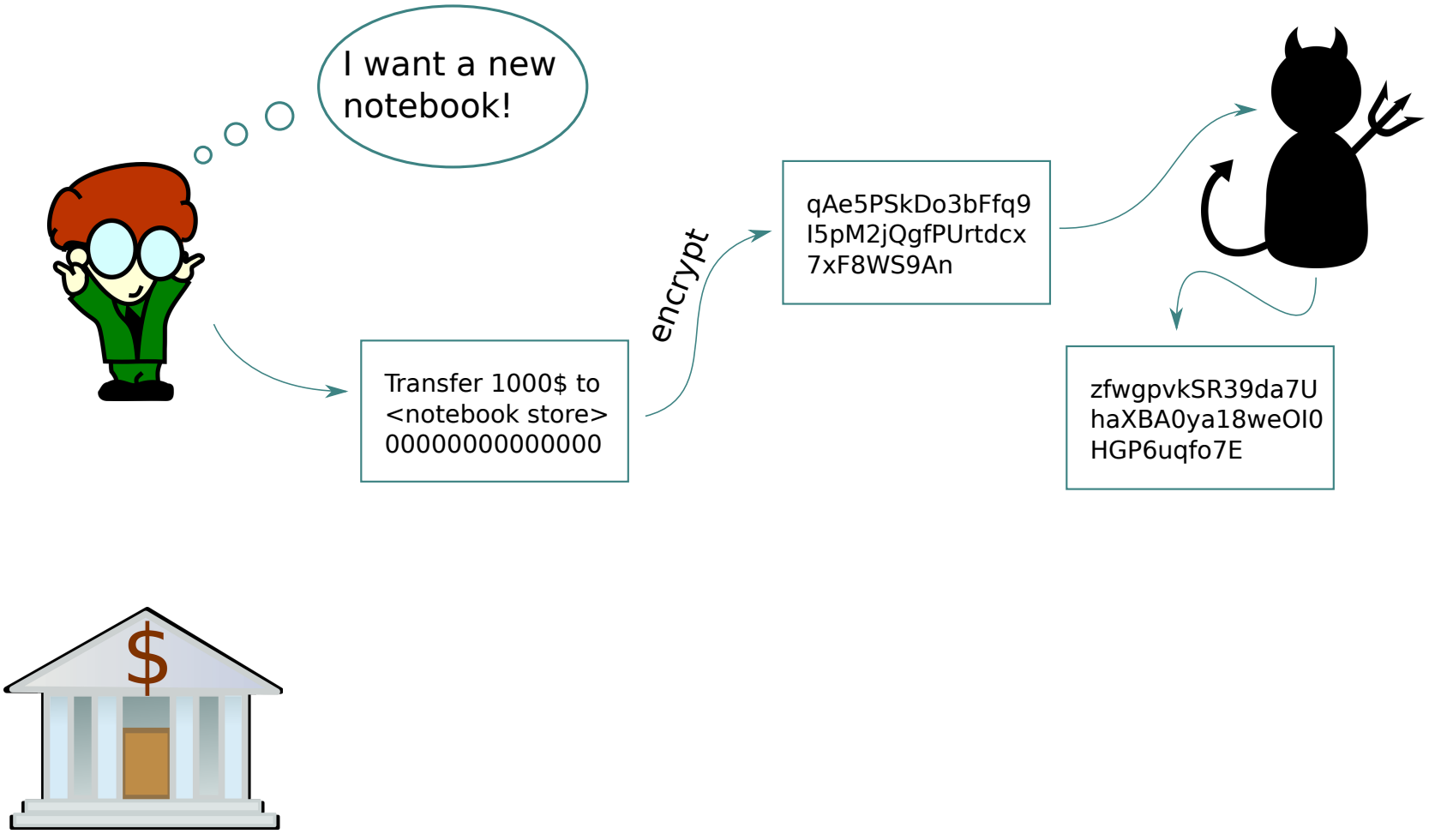


Transfer 1000$ to <notebook store> 00000000000000

encrypt

qAe5PSkDo3bFfq9 I5pM2jQgfPUrtdcx 7xF8WS9An

Authentication from NM: Intuition



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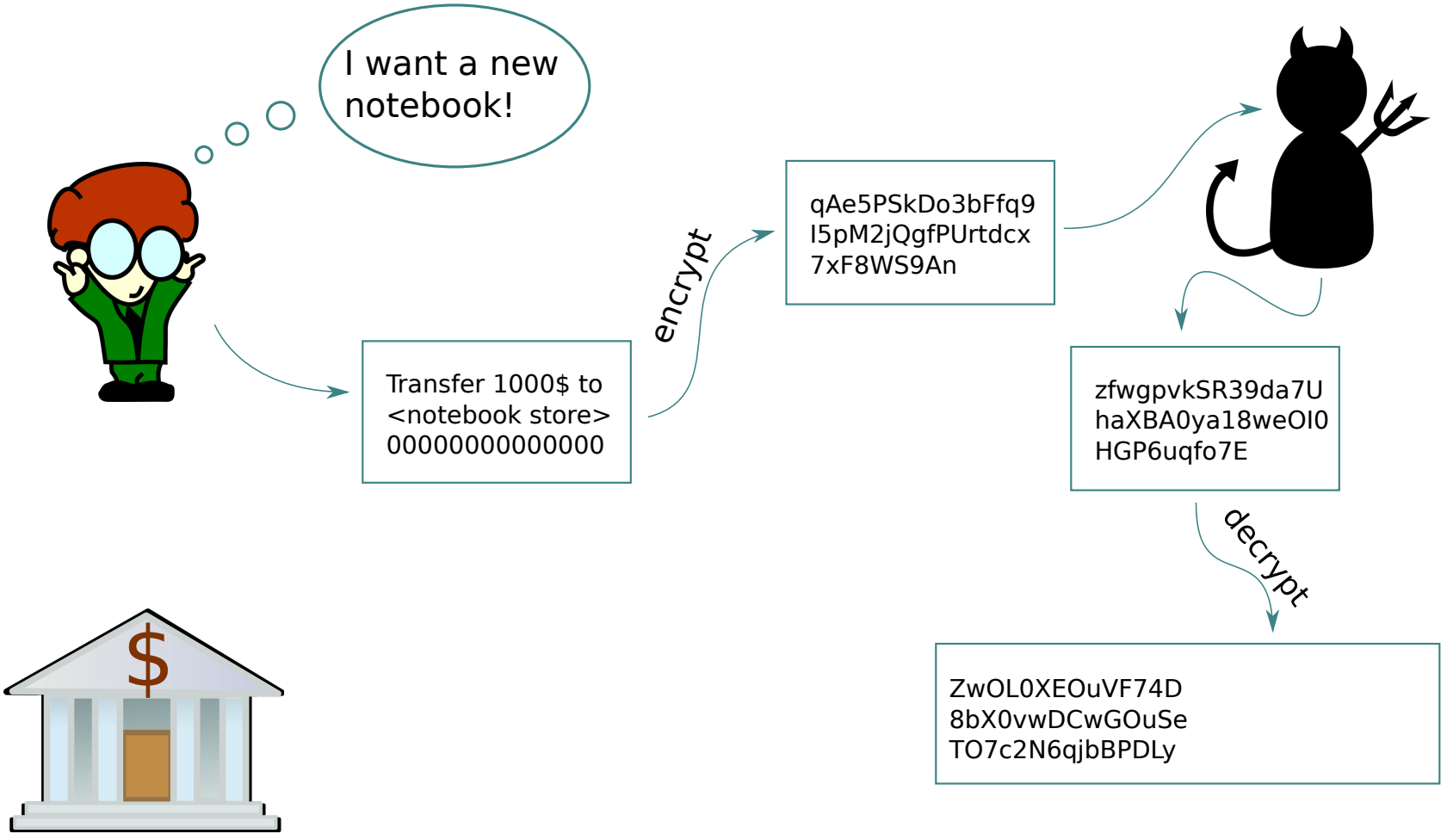

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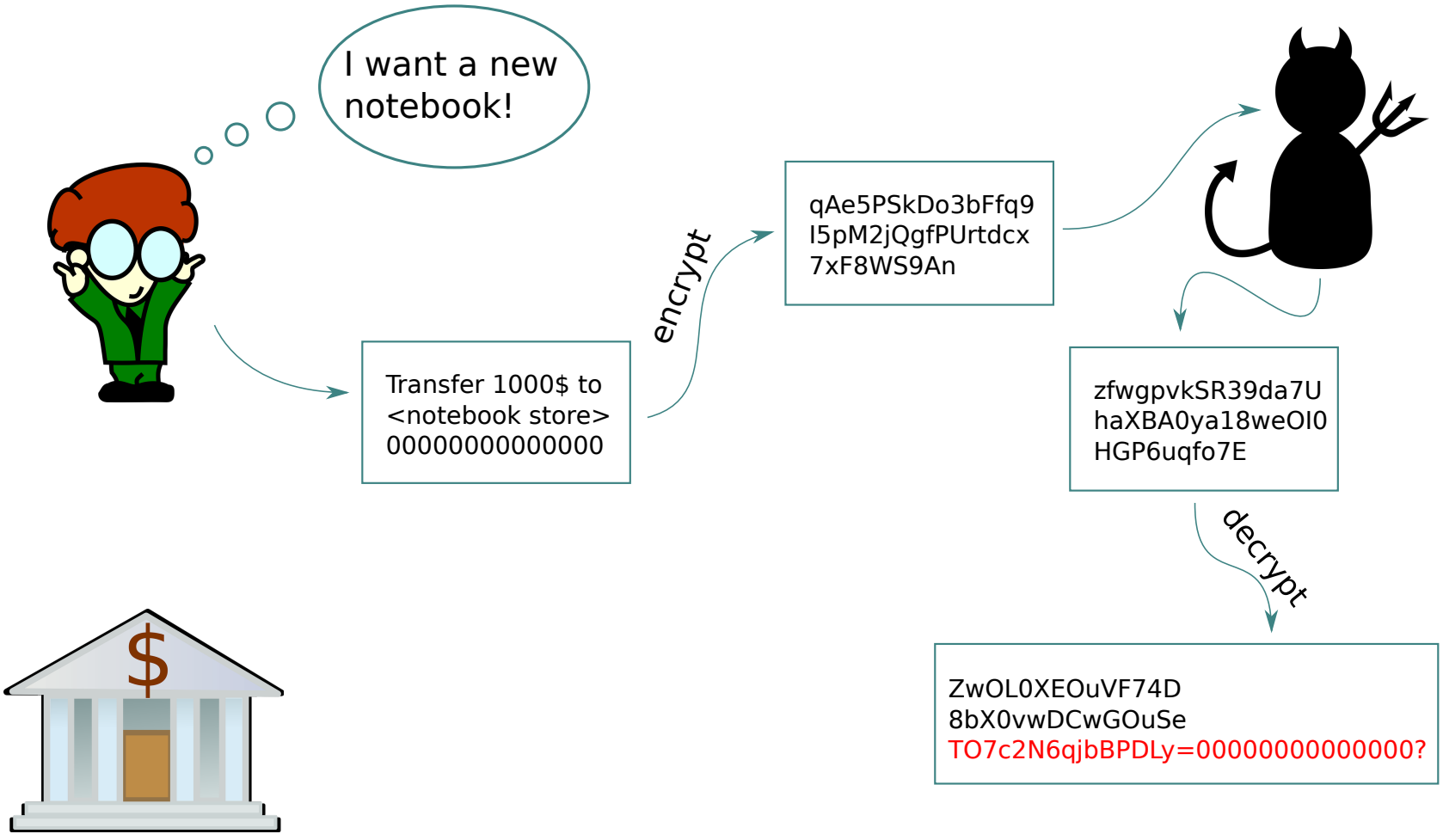
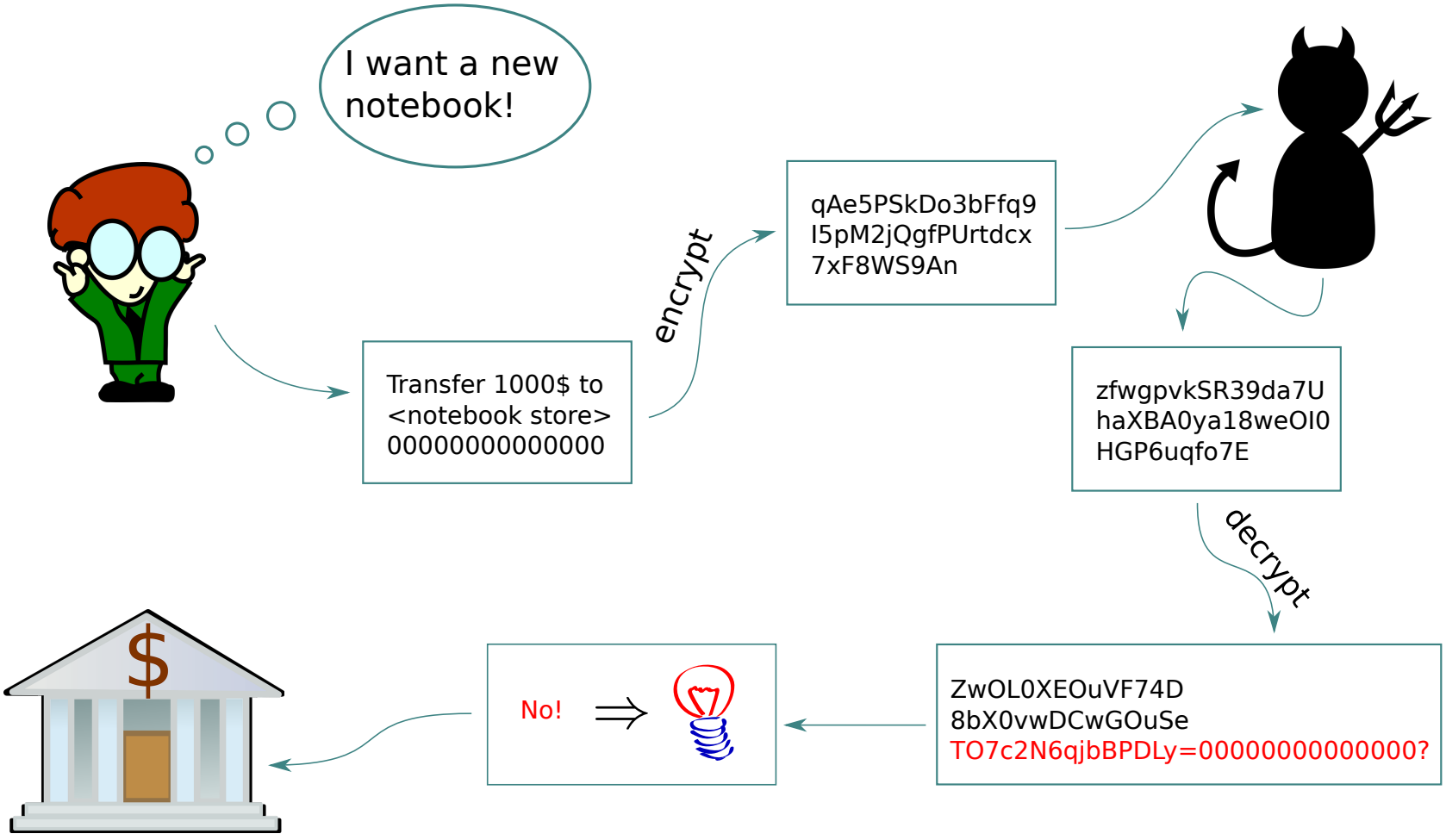


Authentication from NM: Intuition



Authentication from qNM

Theorem (Alagic, CM)

Adding a constant tag to a quantum message and encrypting it with an qNM scheme achieves DNS-authentication

Summary authentication

- ✓ DNS authentication from qNM schemes via tagging
- ✓ GYZ authentication from 2-designs instead of 8-designs

Open questions

Computational security?

Current work with Gorjan Alagic and Tommaso Gagliardoni

Can we improve the Λ-dependence of NM?

NM with high probability?