Security of the Fiat-Shamir Transformation in the Quantum Random-Oracle Model

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and Christian Schaffner

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University of Amsterdam

Universiteit Leiden

NWO

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Fiat-Shamir reconciles the two in certain cases.

Outline

1. Introduction
   - Interactive proof systems
   - The Fiat Shamir transformation
2. Results
   - Overview
   - Reduction
   - Techniques
3. Application: Digital Signatures

1. Introduction

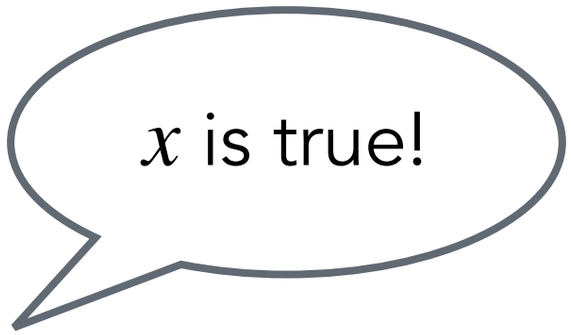
Interactive proof system

Interactive proof system

Prover

Verifier

Interactive proof system



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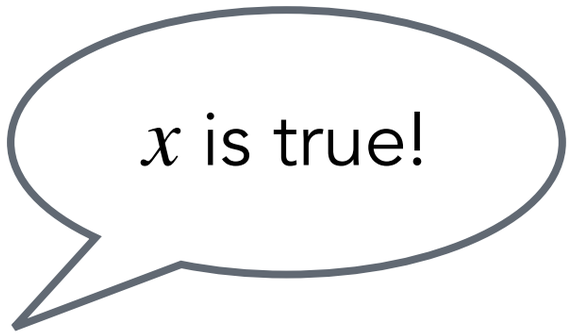

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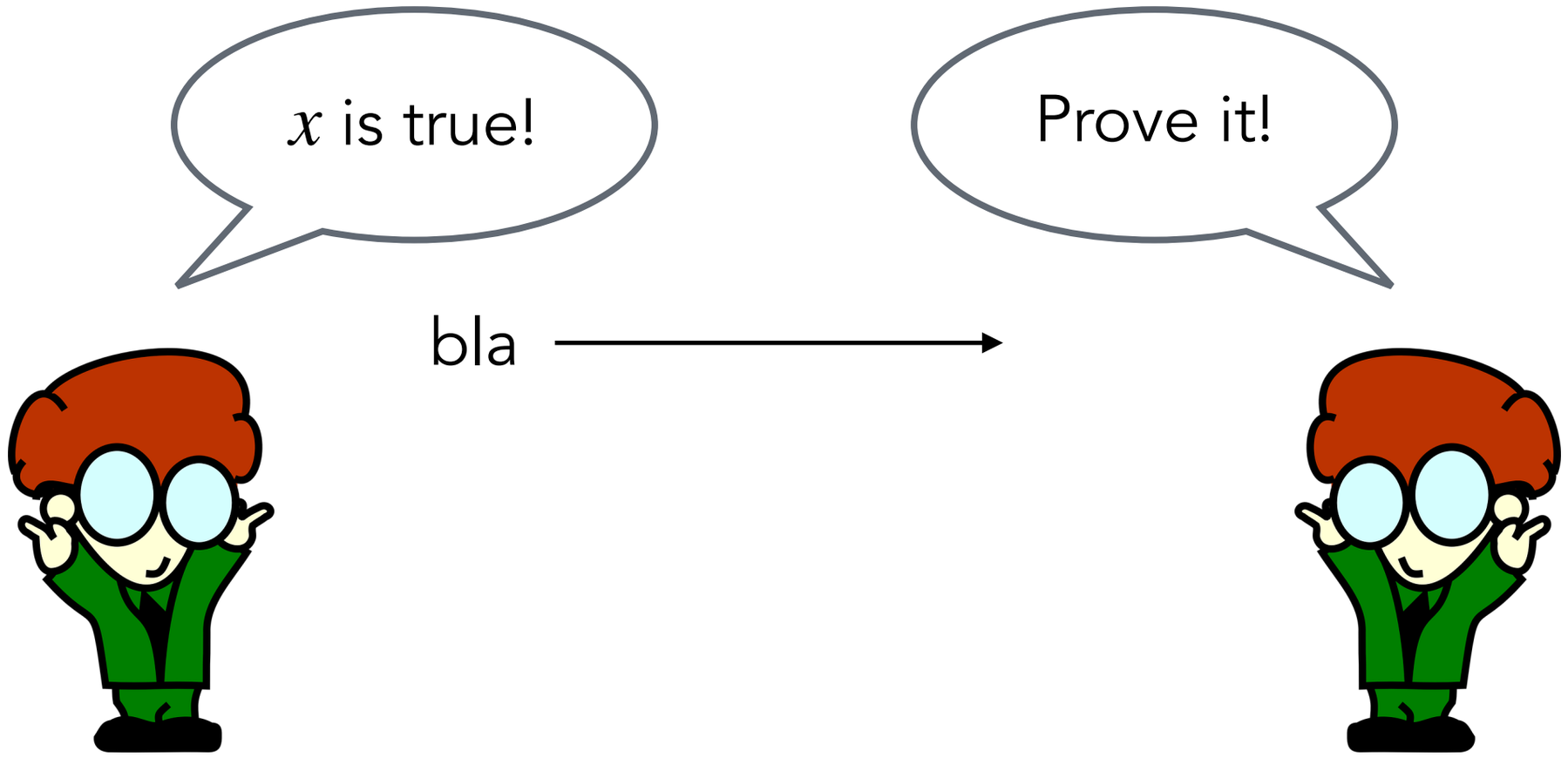

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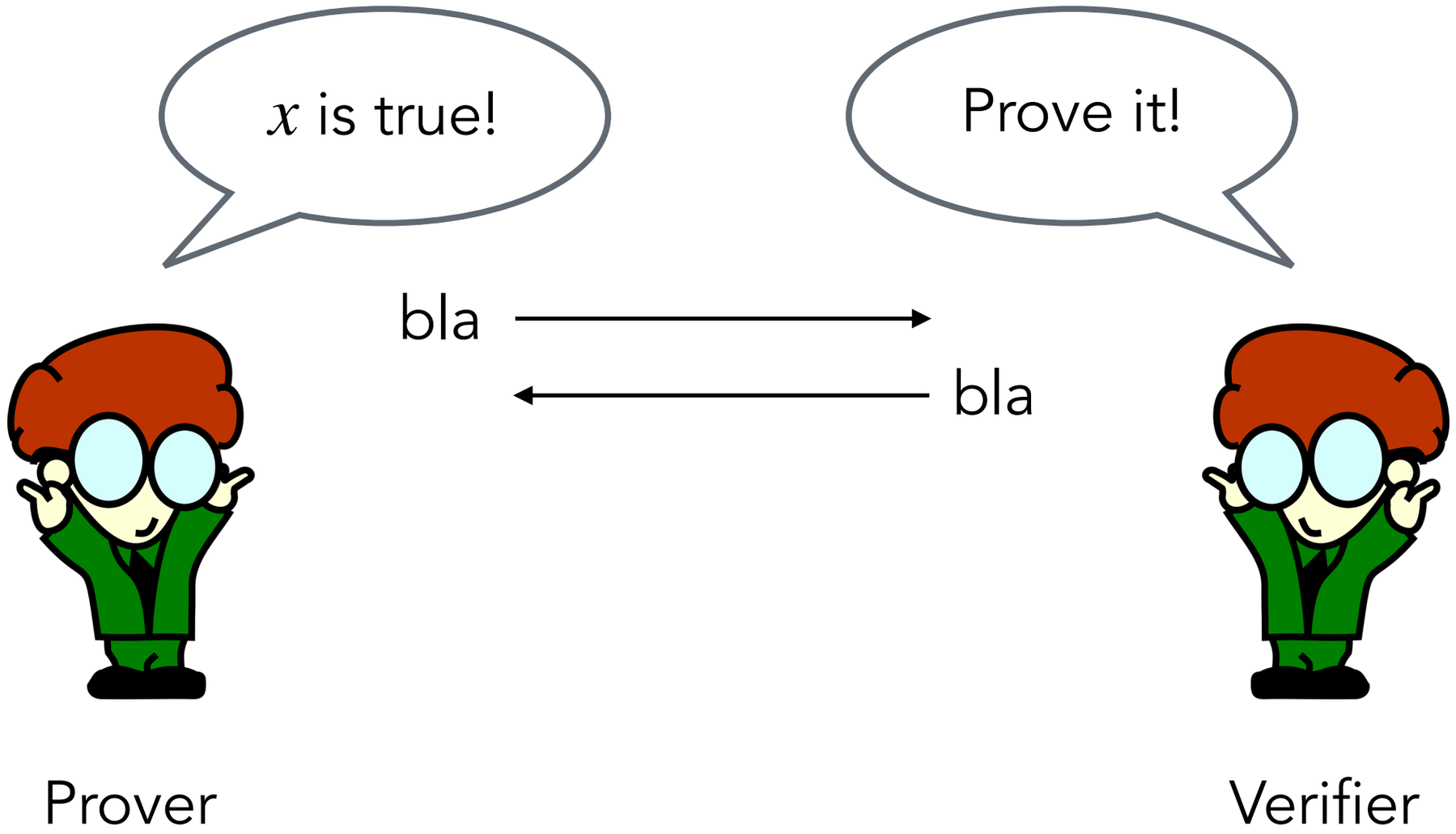

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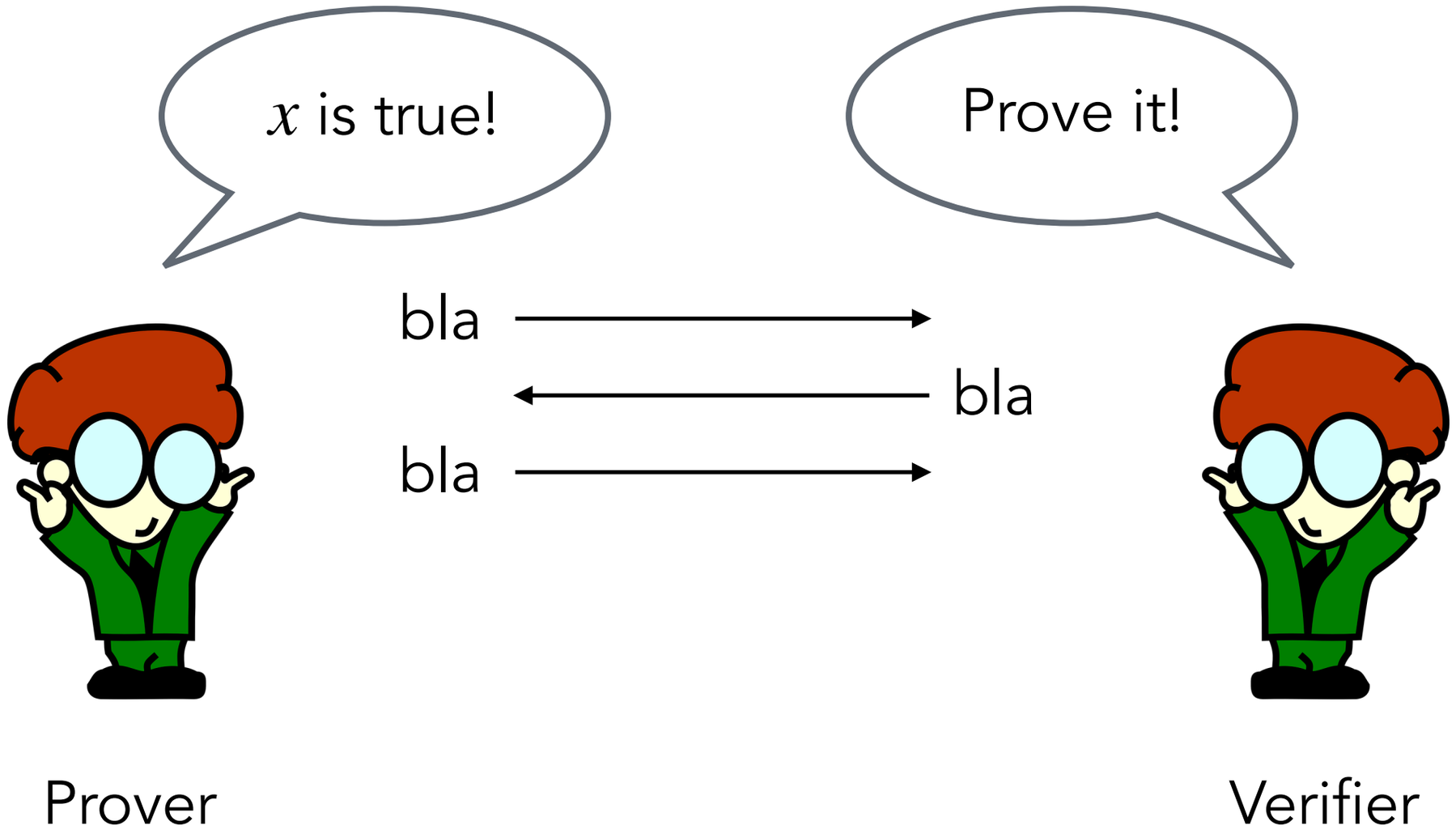

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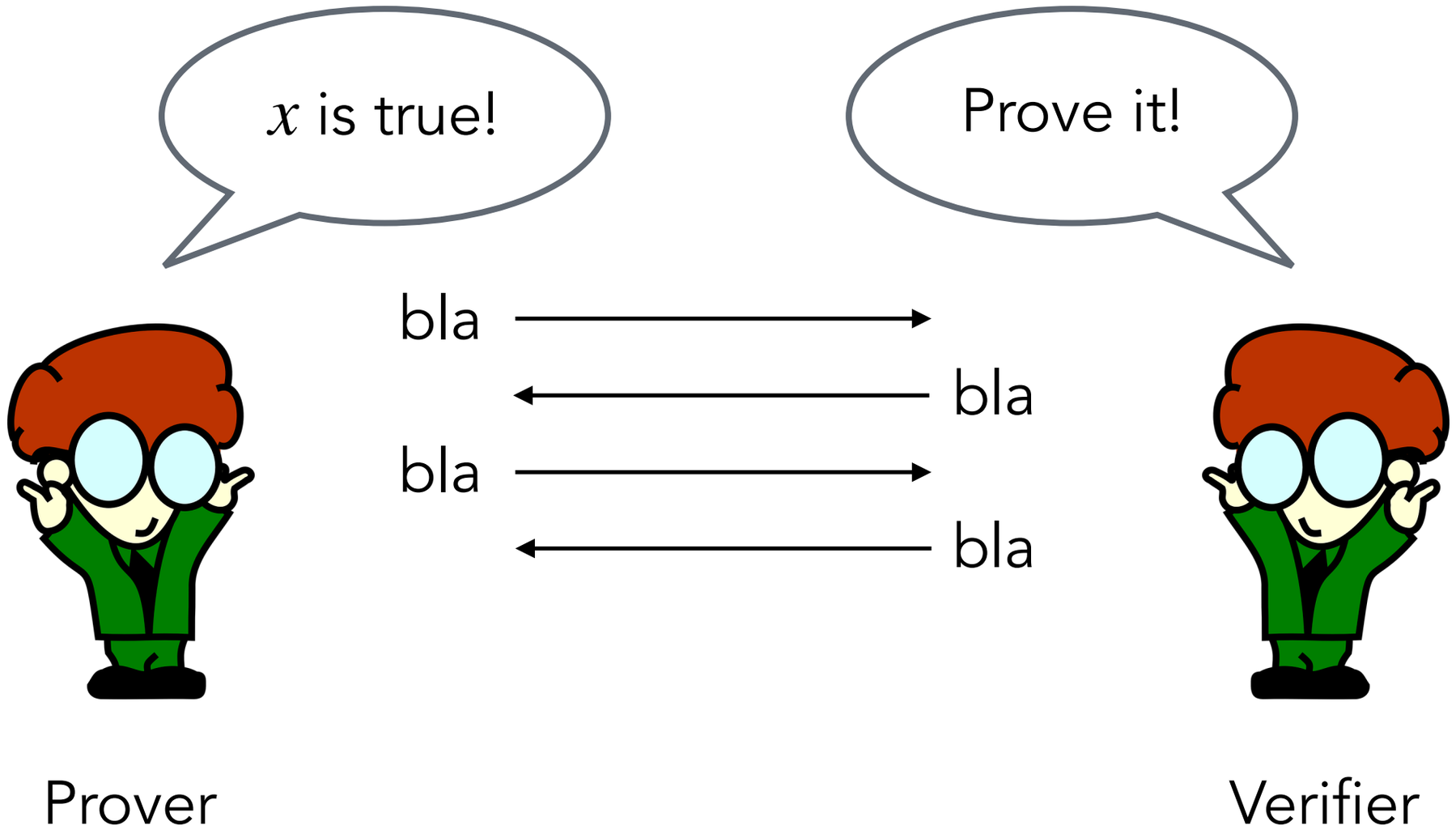

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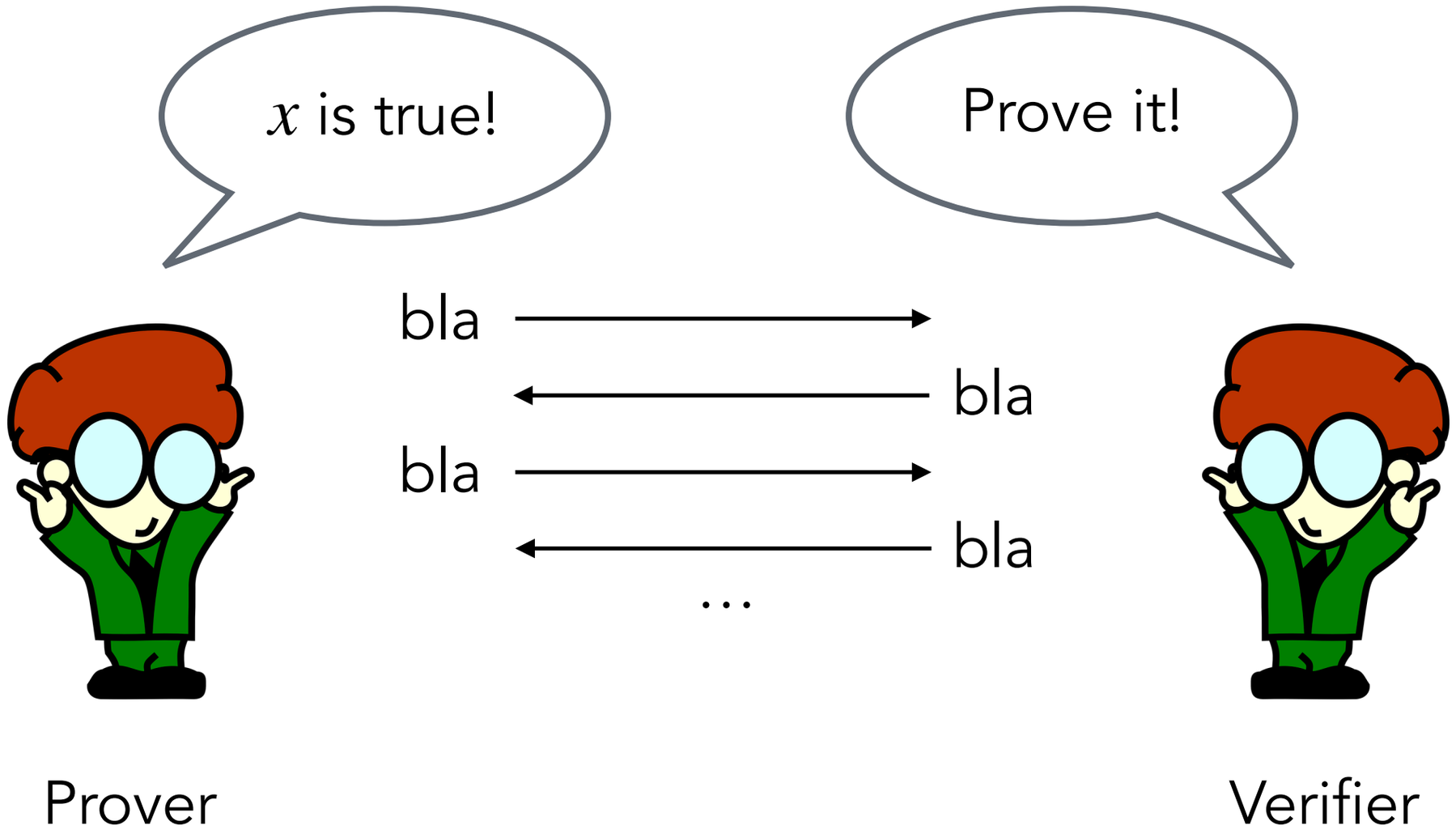

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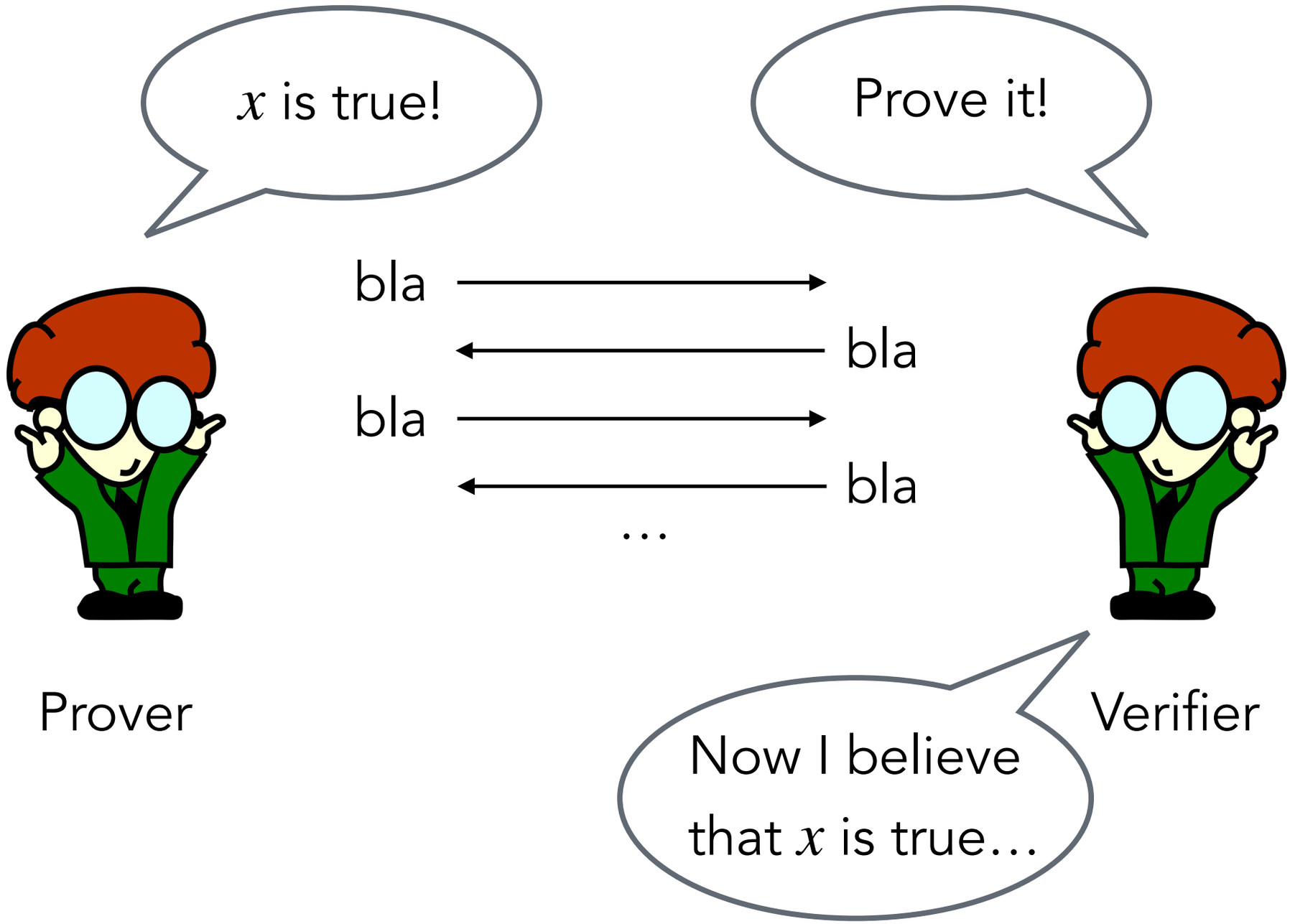

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Yes, at least in some cases, using the Fiat Shamir transformation

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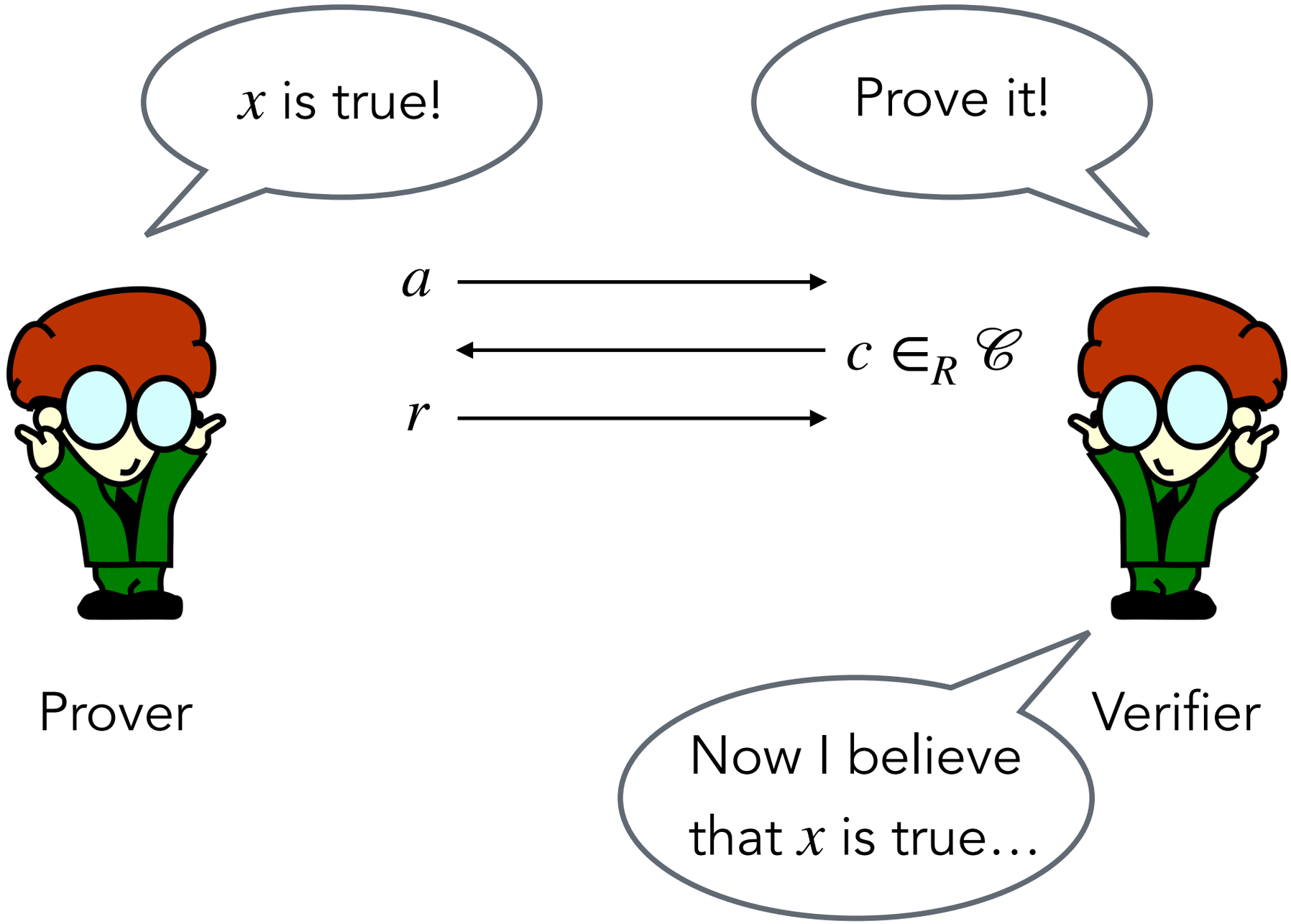
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Prove it!
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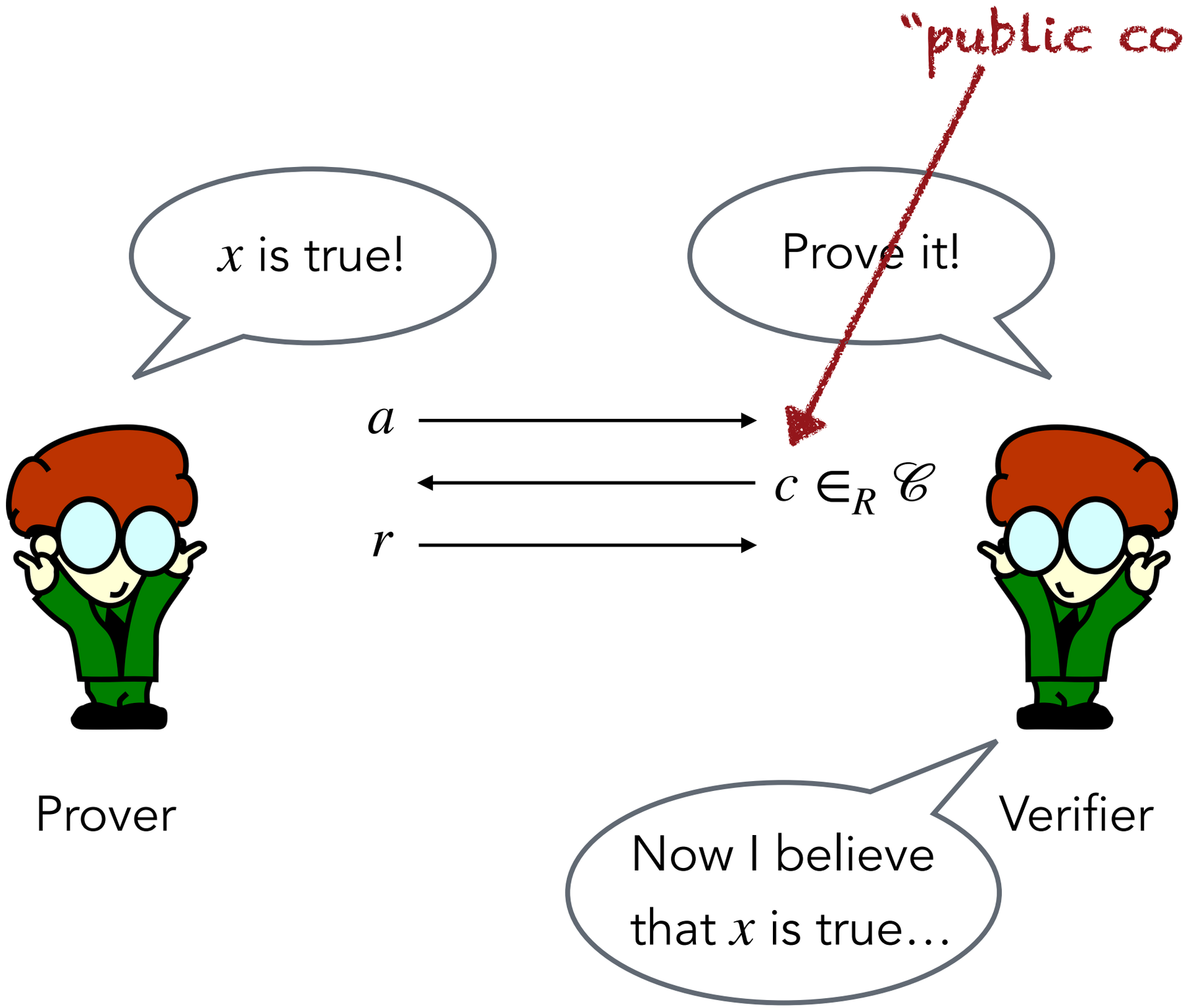
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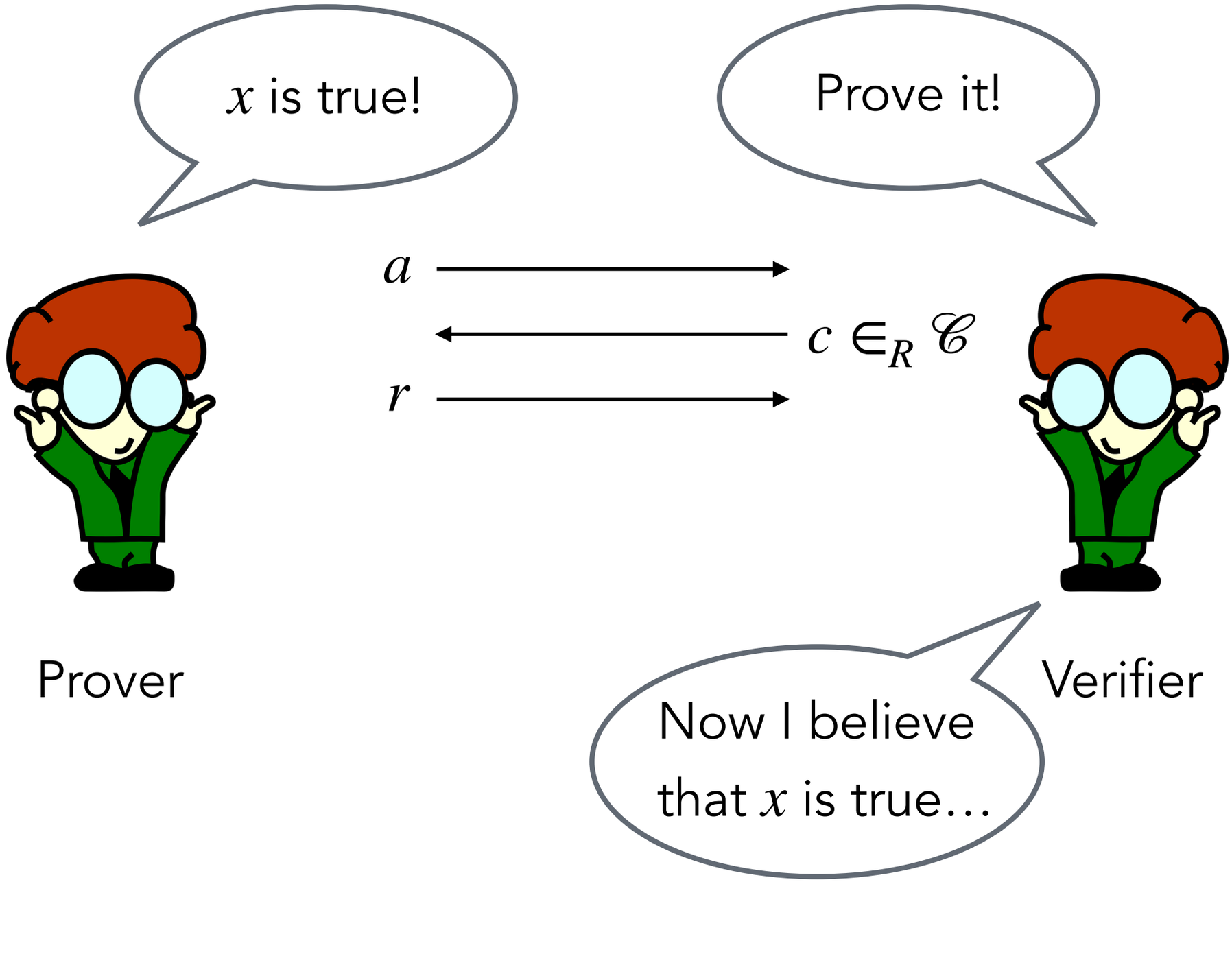

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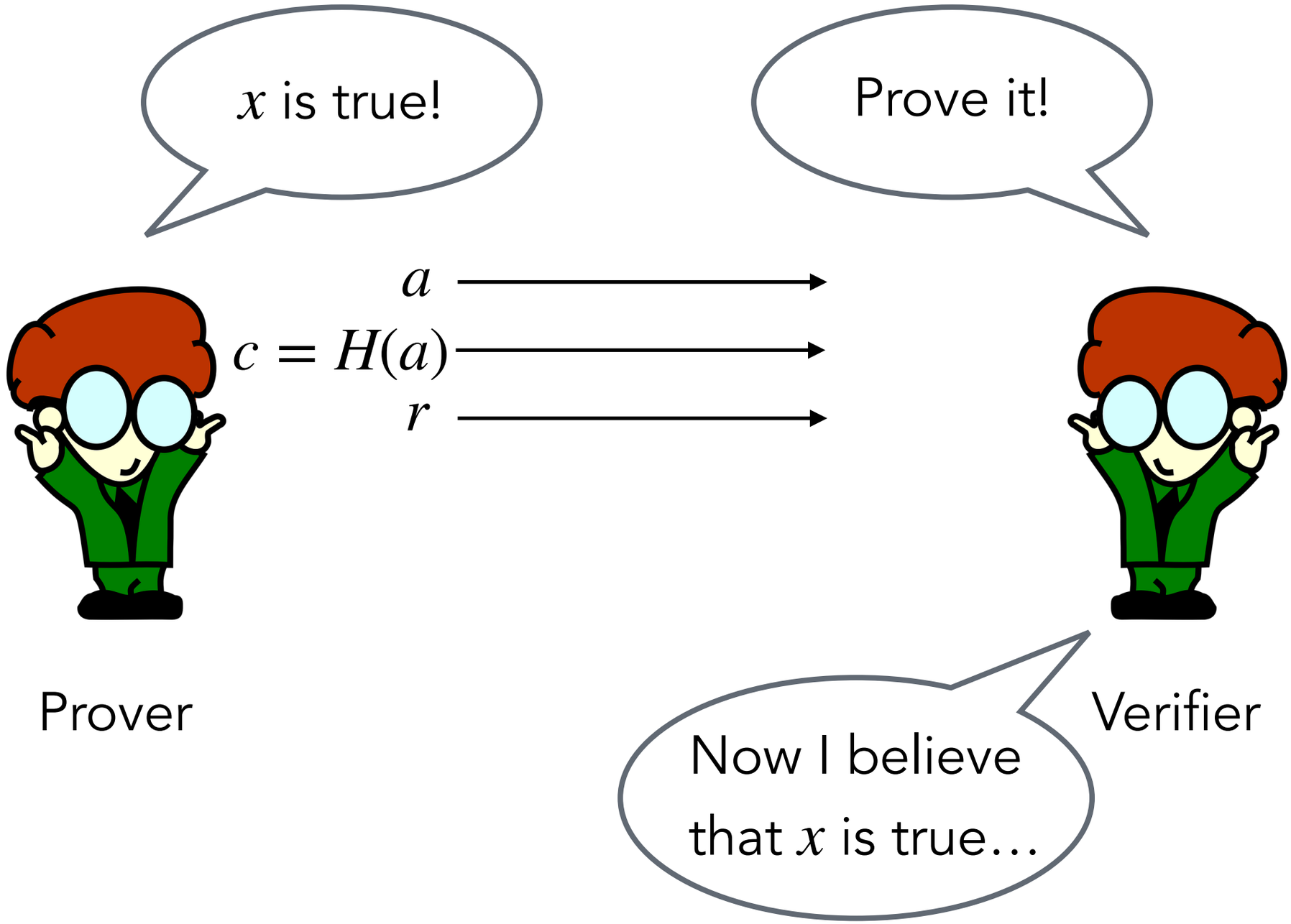

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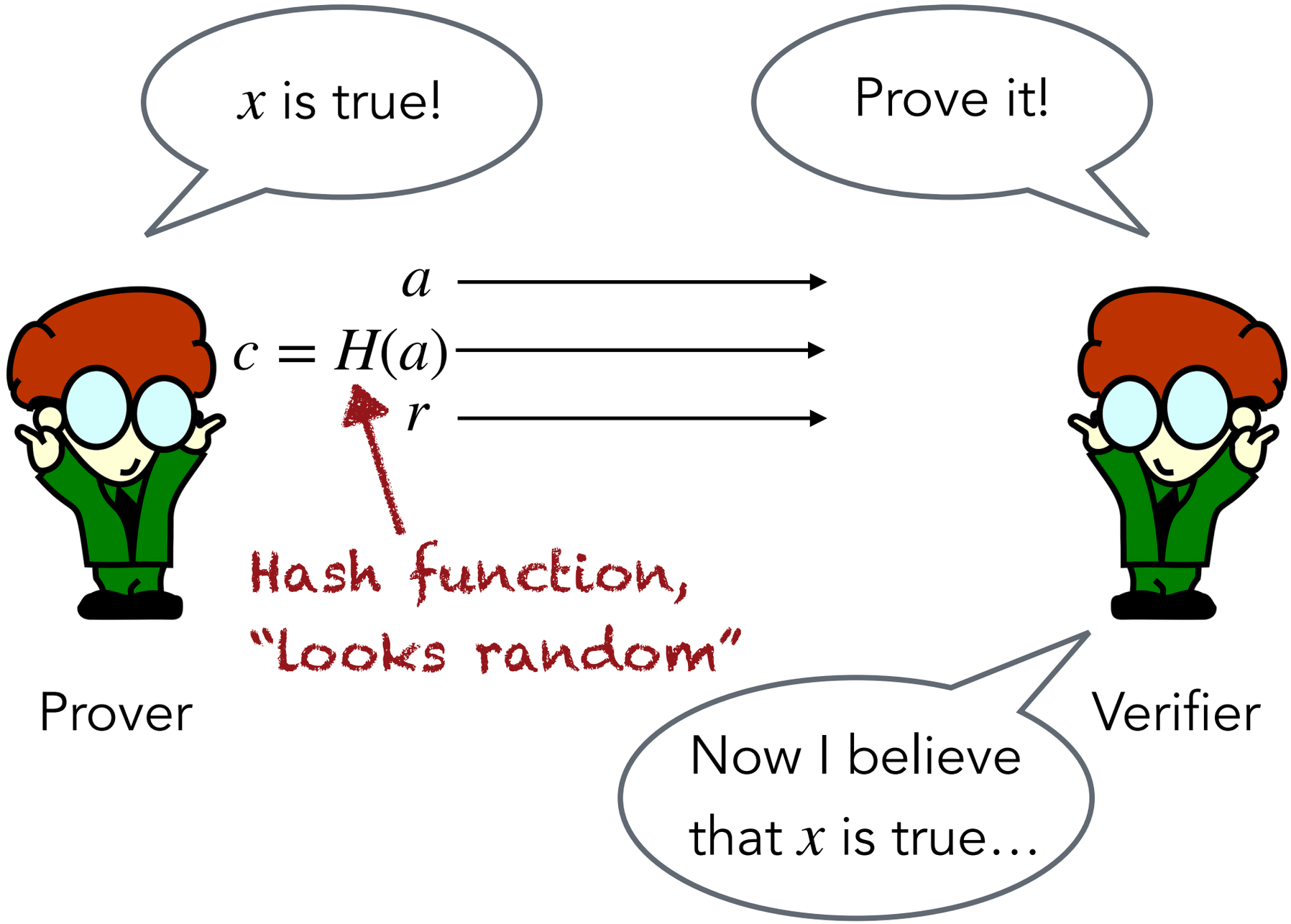

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Pretend that hash function is random and everybody has oracle access

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Many cases important for post-quantum crypto still open.

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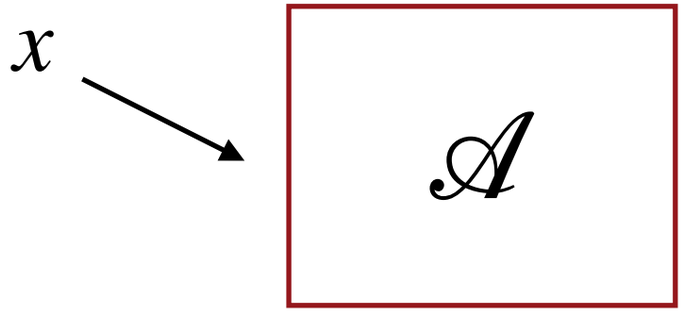

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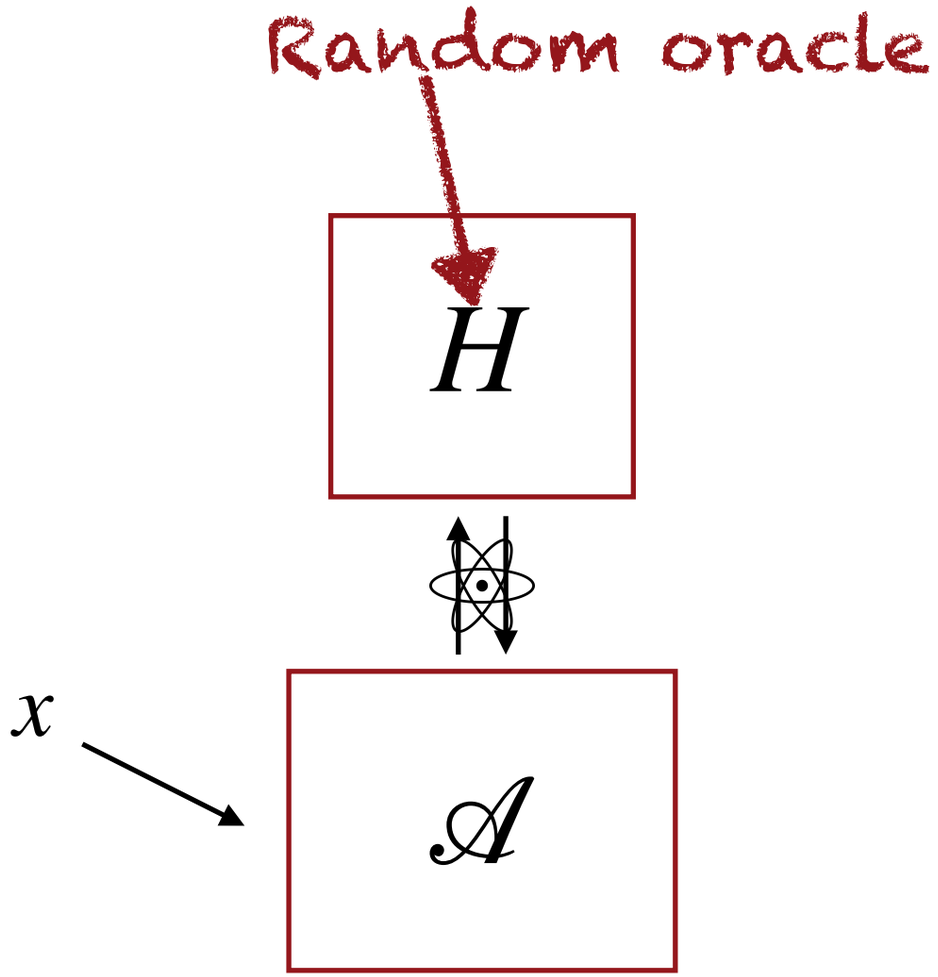

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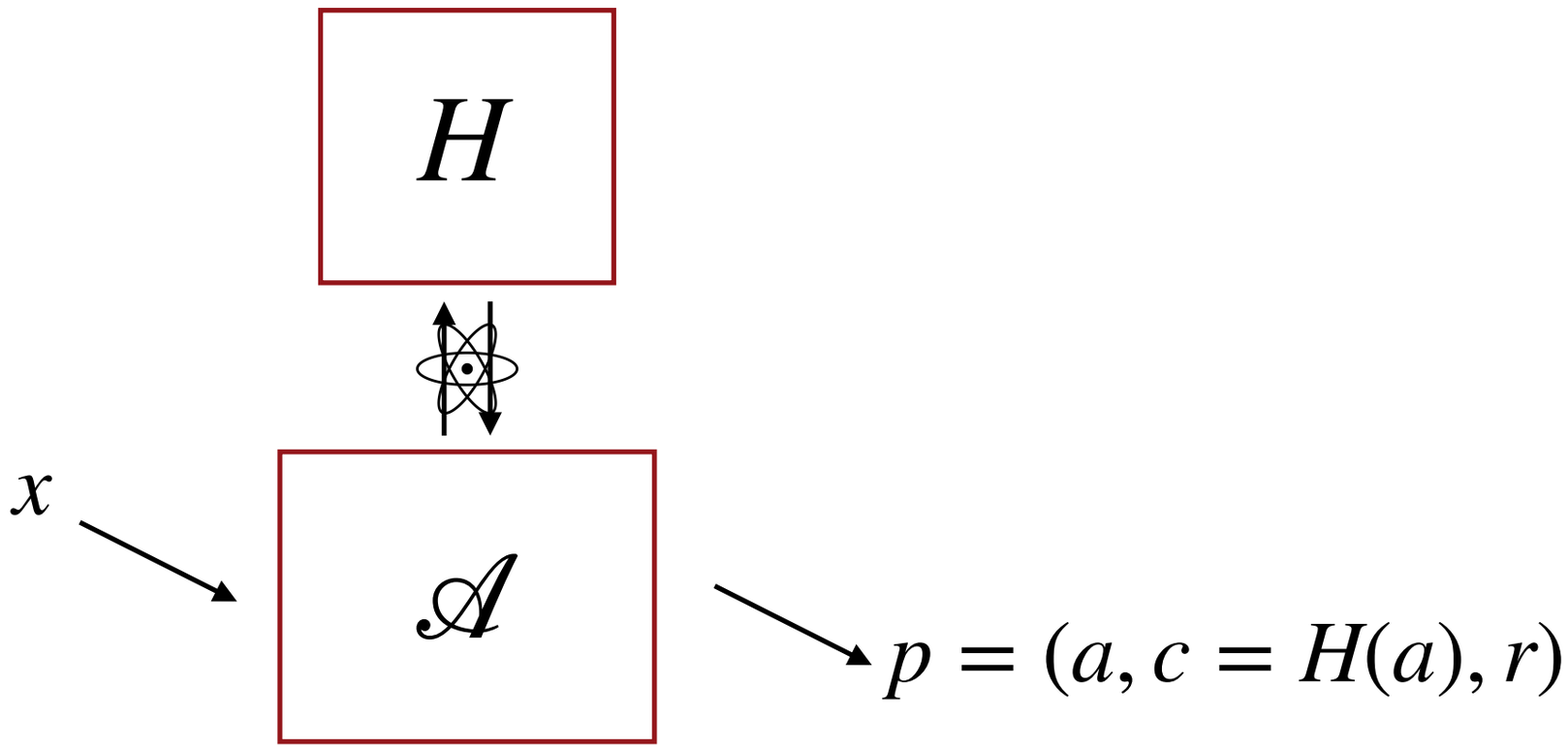

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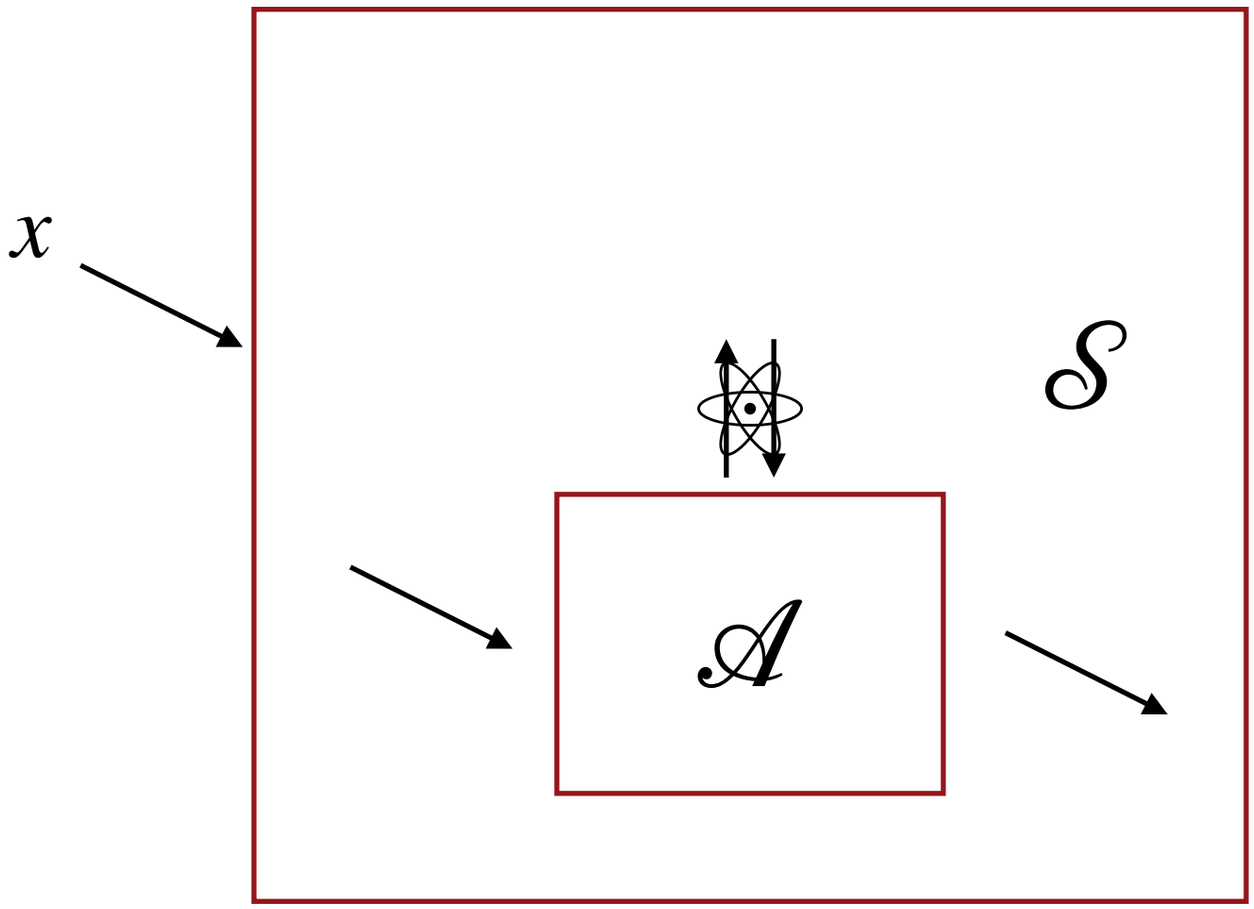

Verifier

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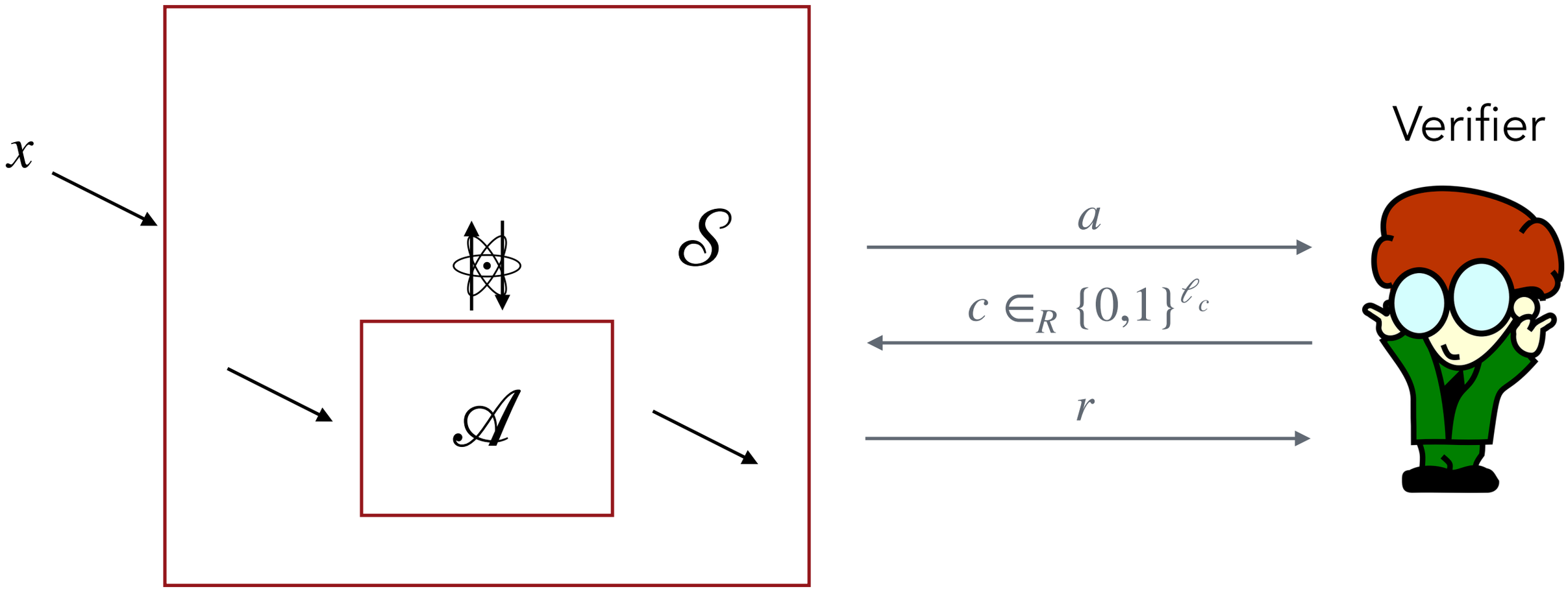

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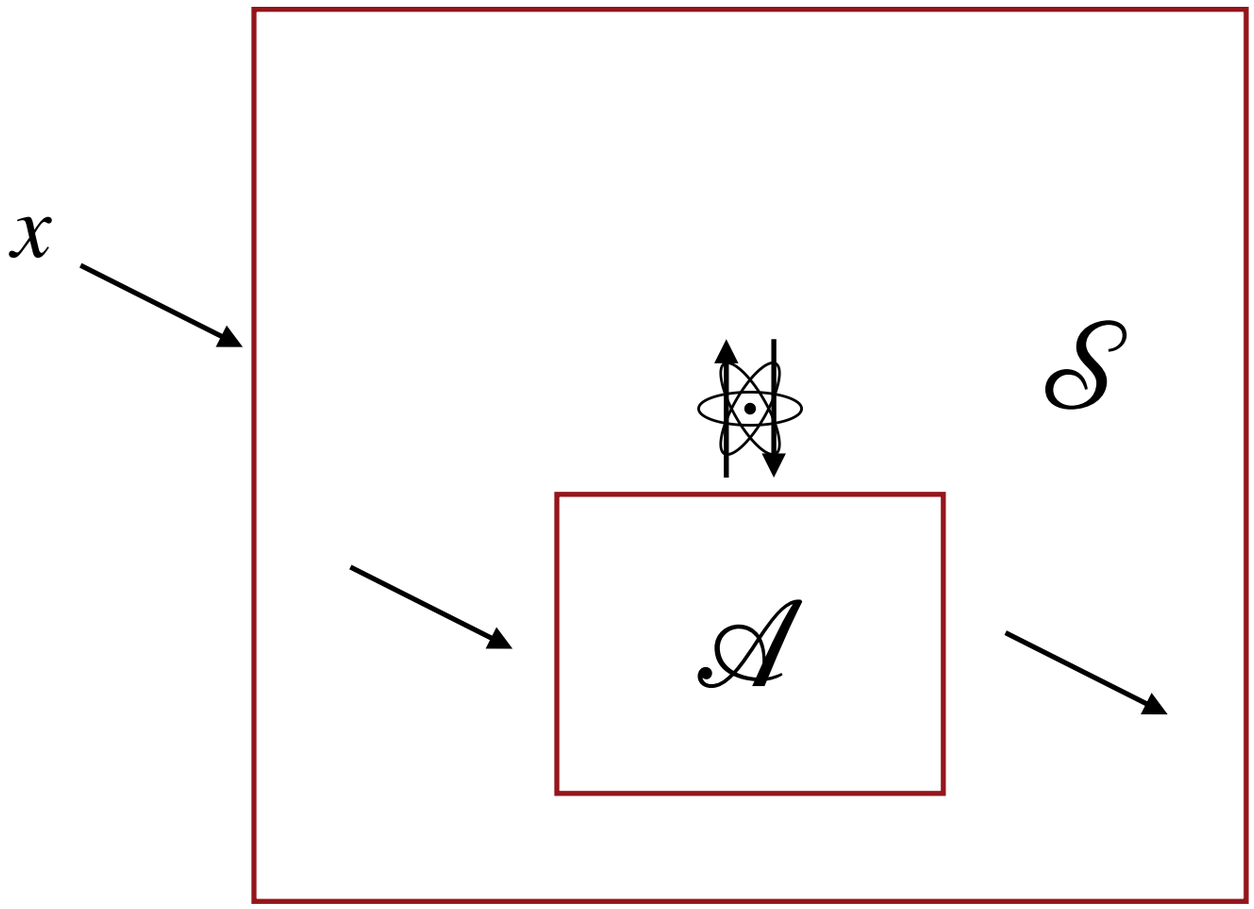

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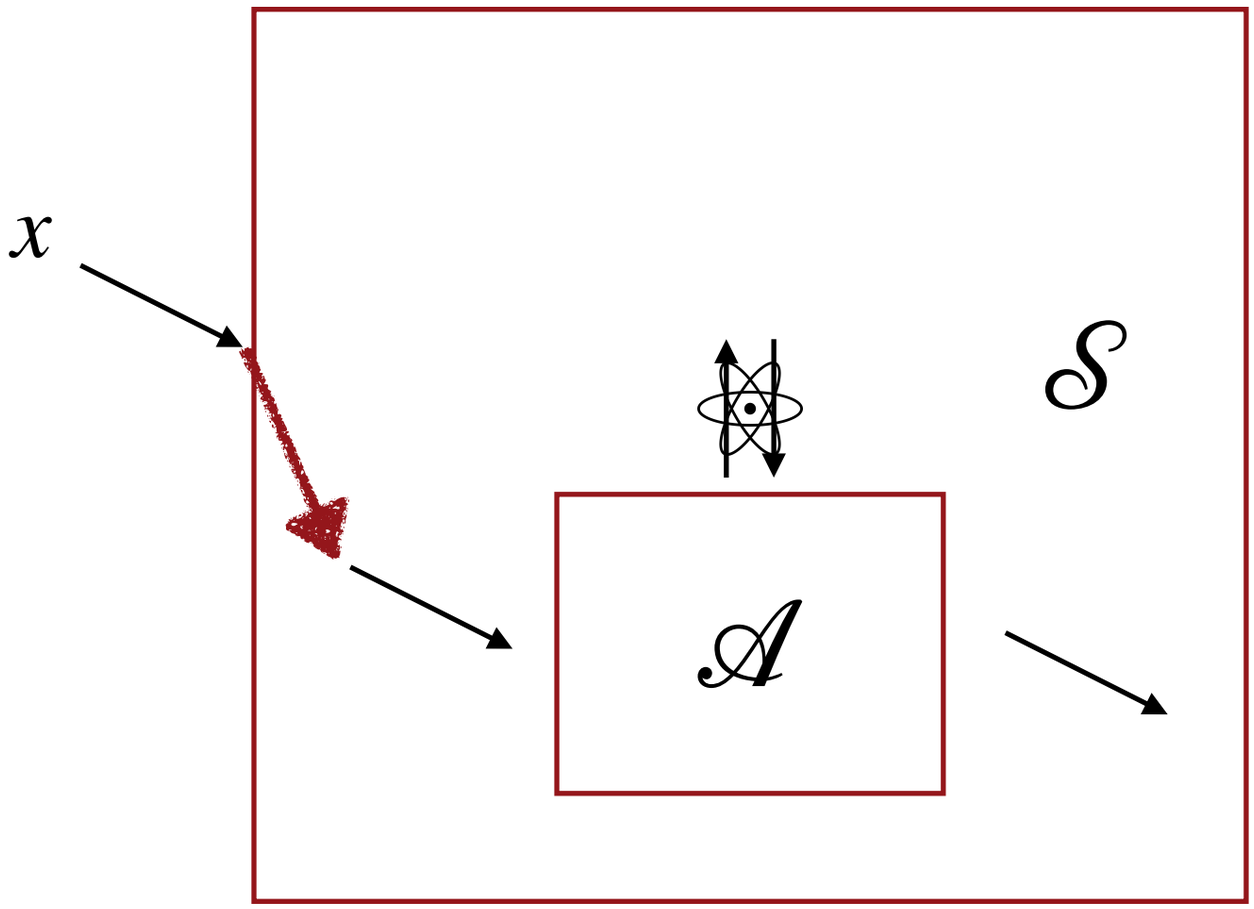

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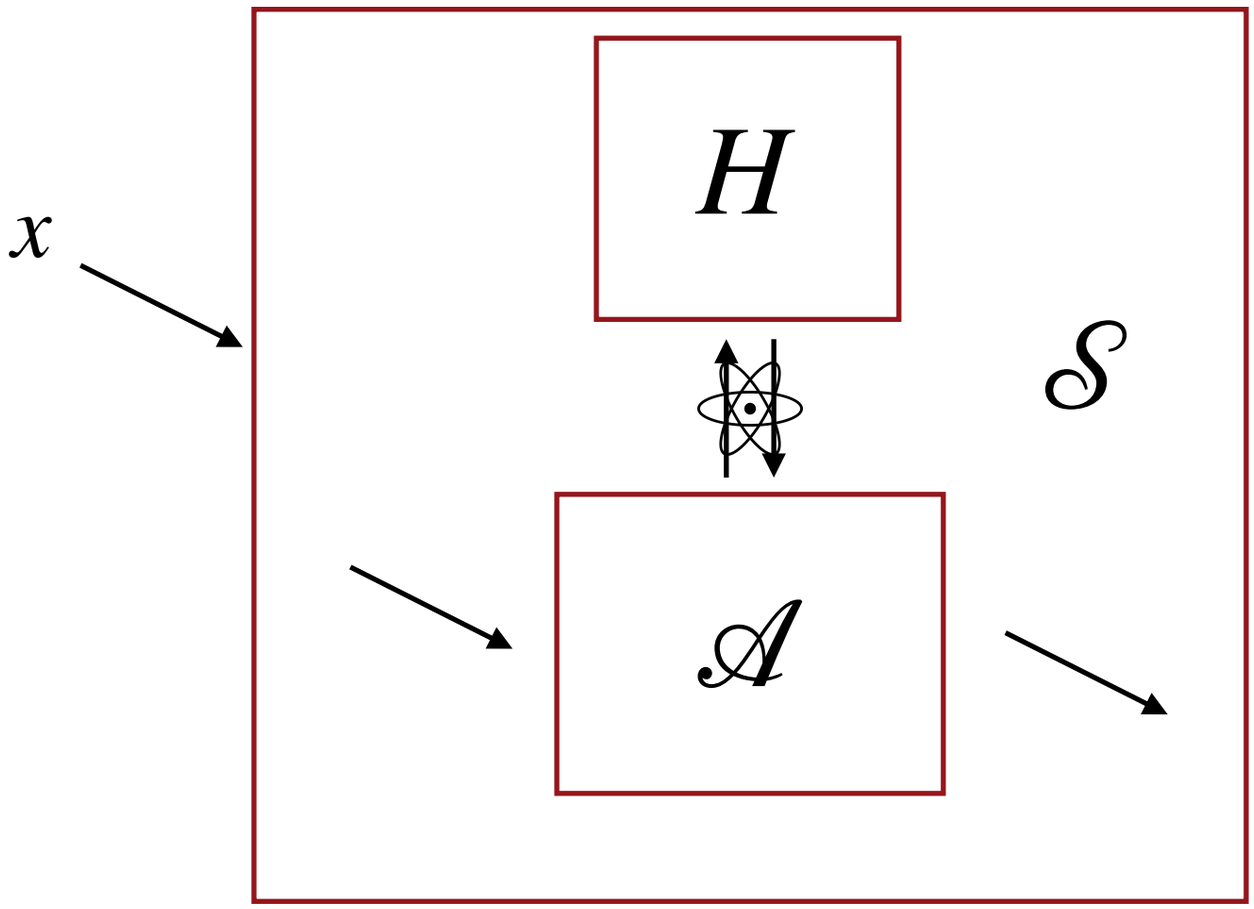

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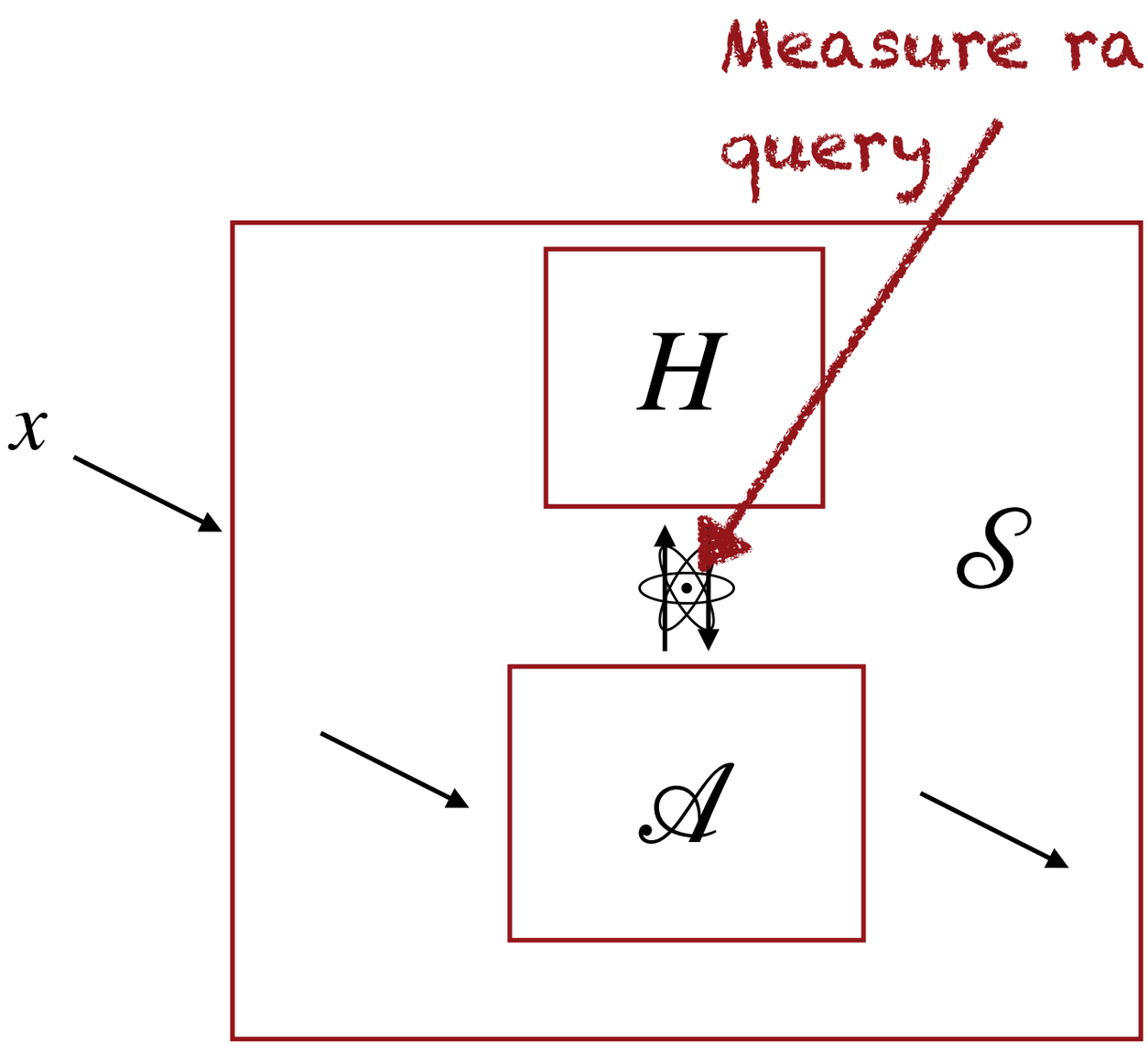

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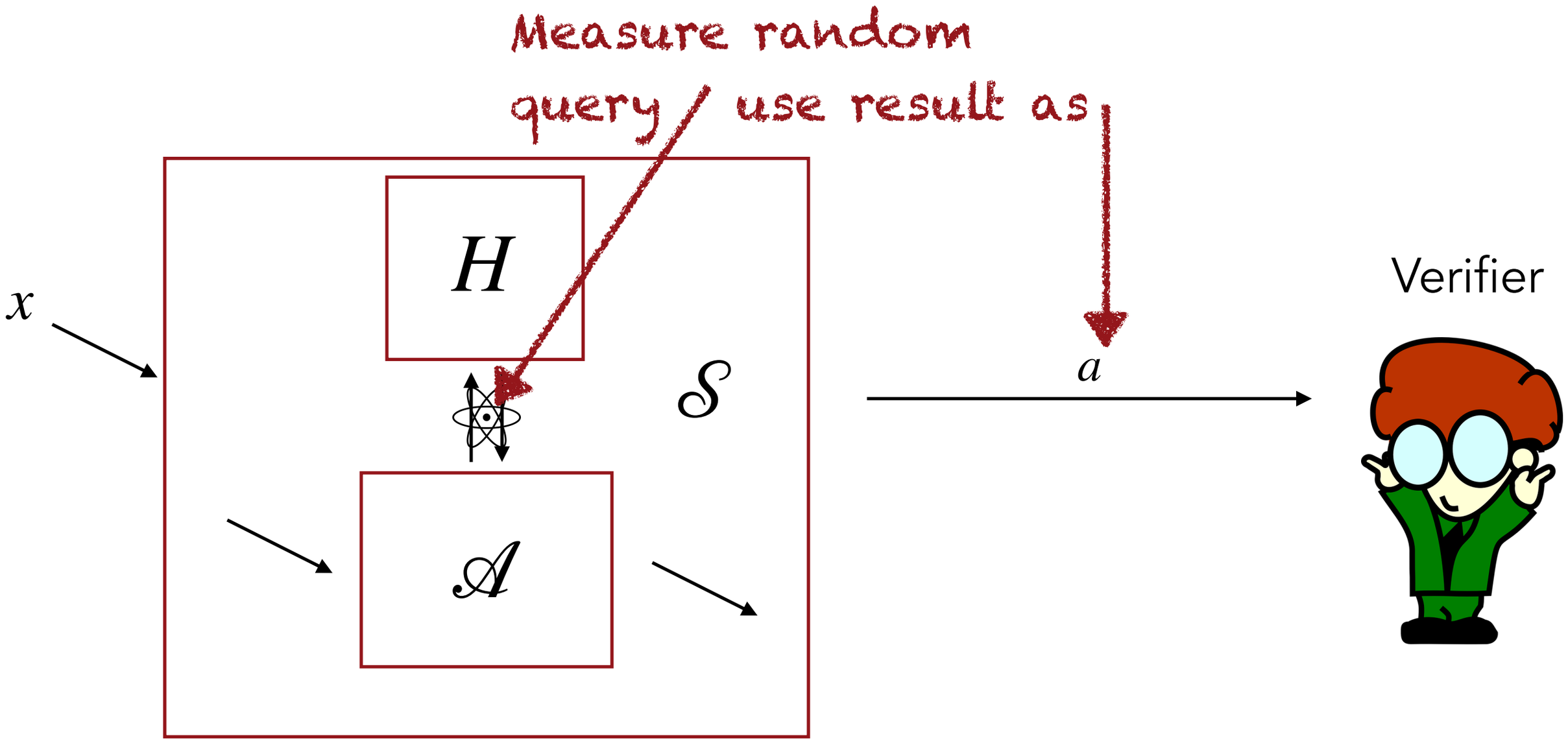

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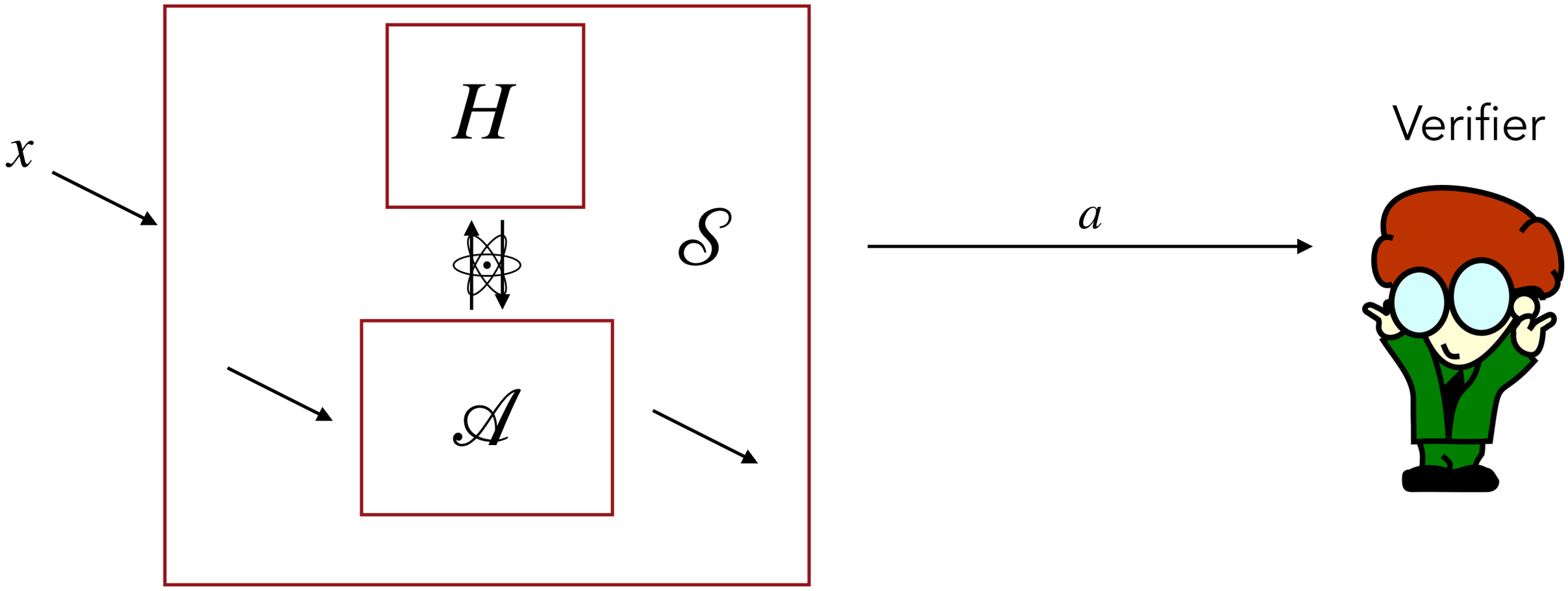

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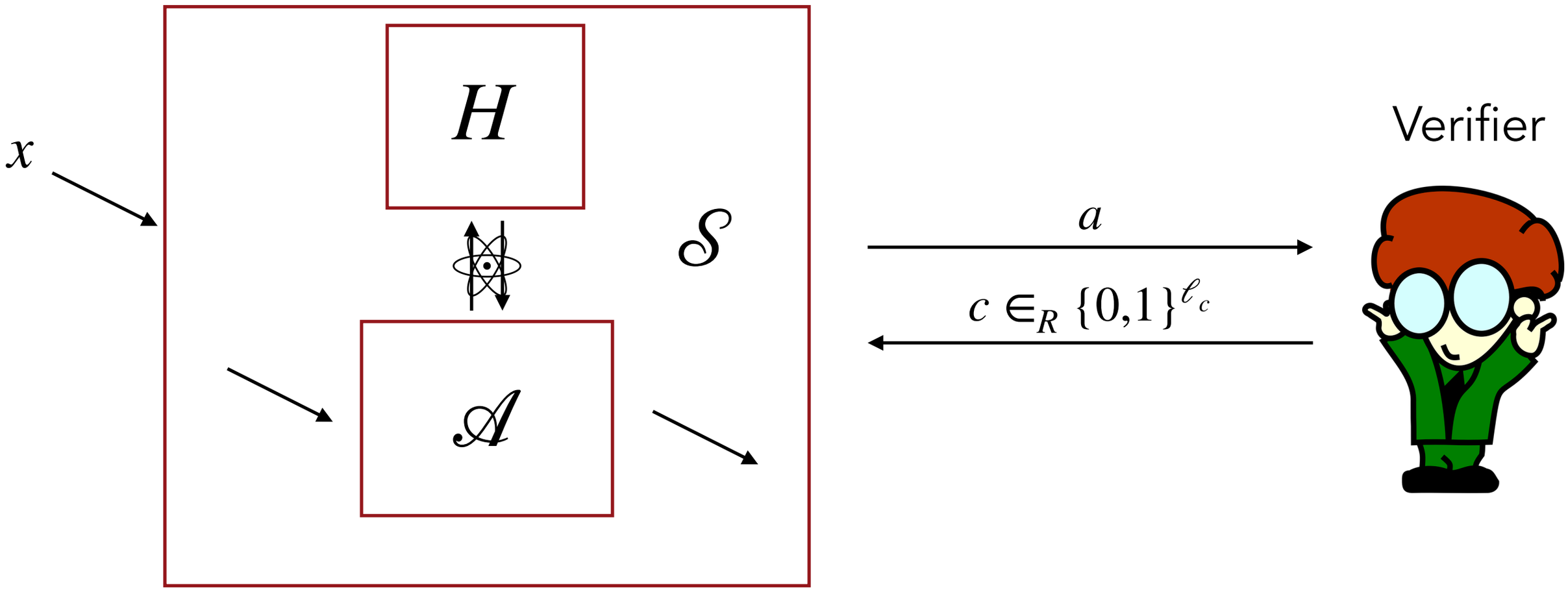

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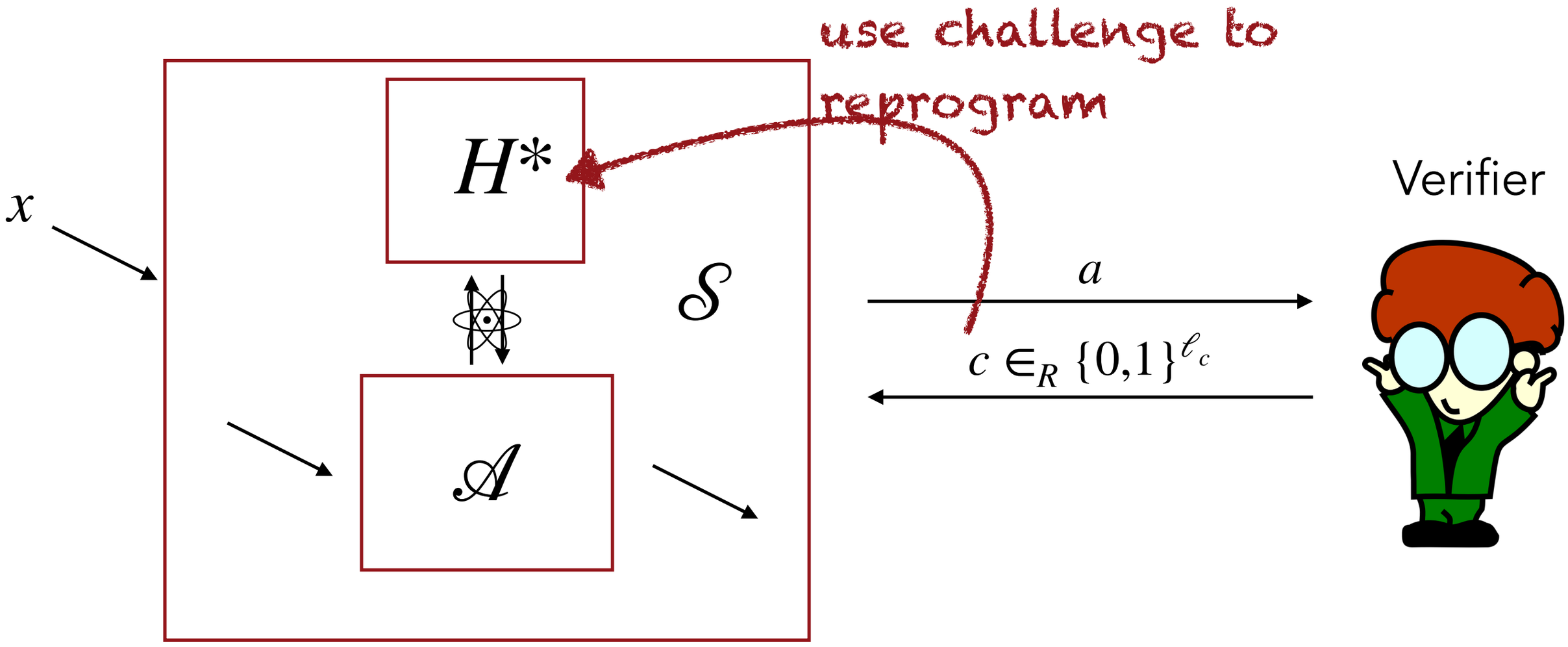

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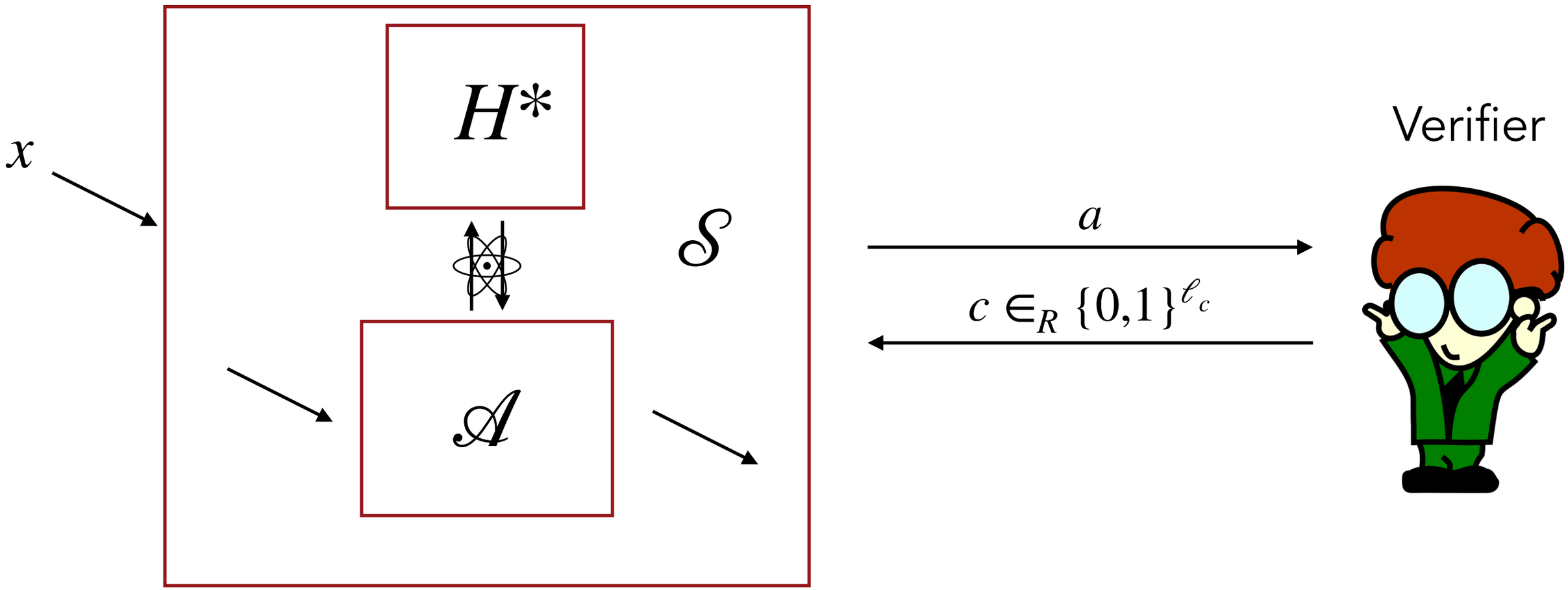

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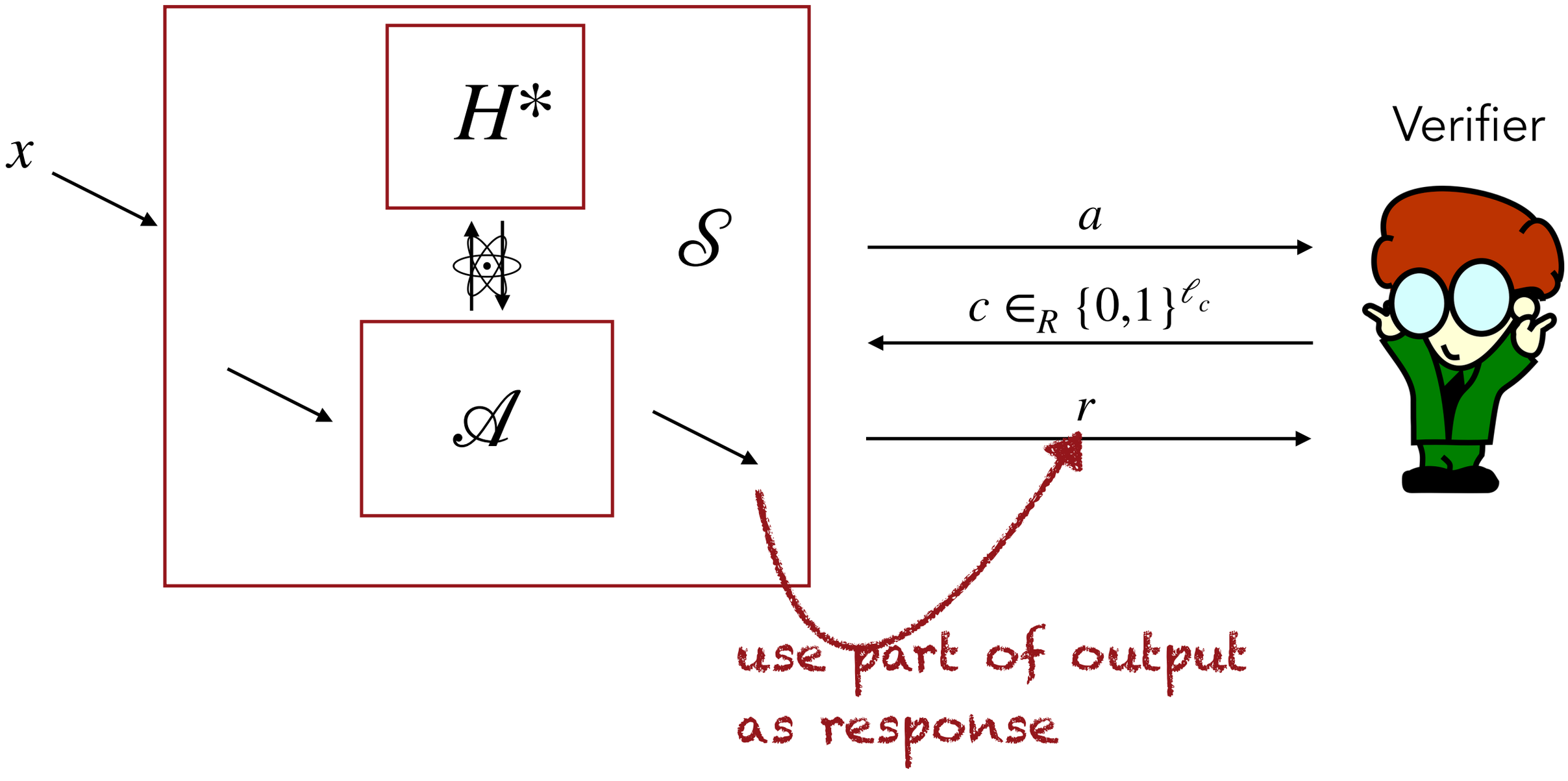

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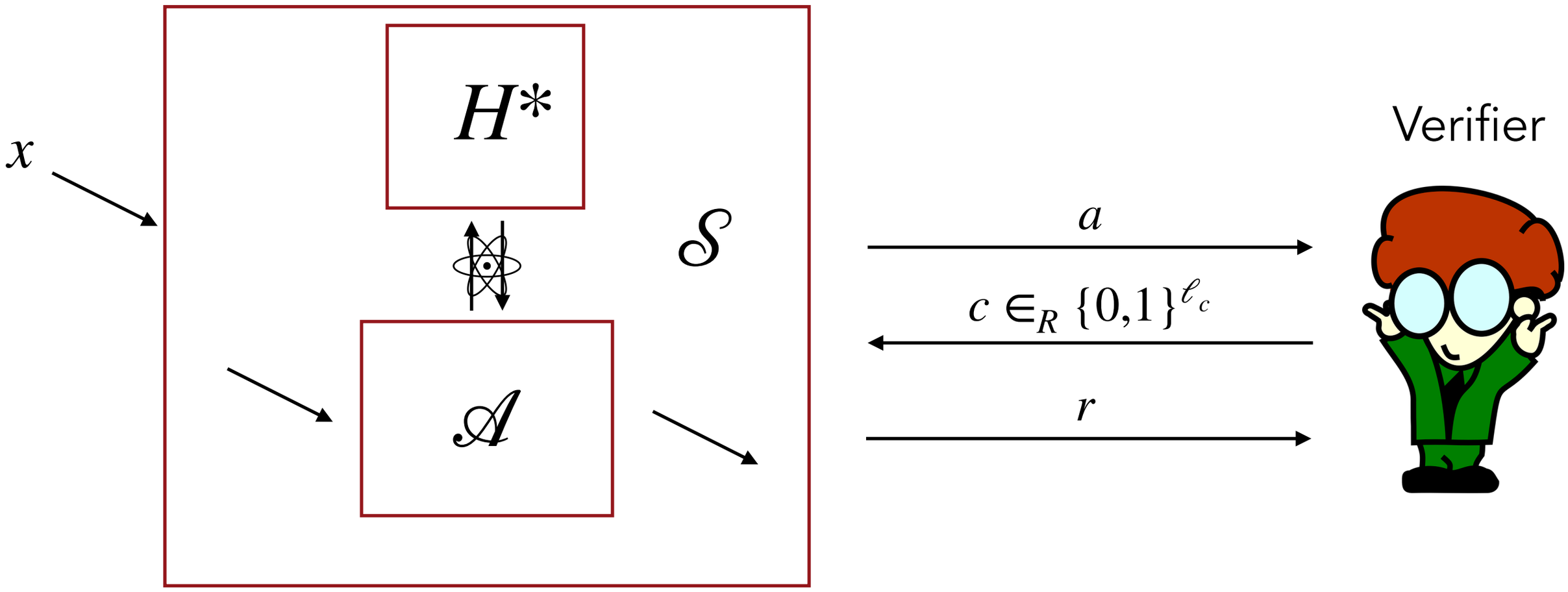

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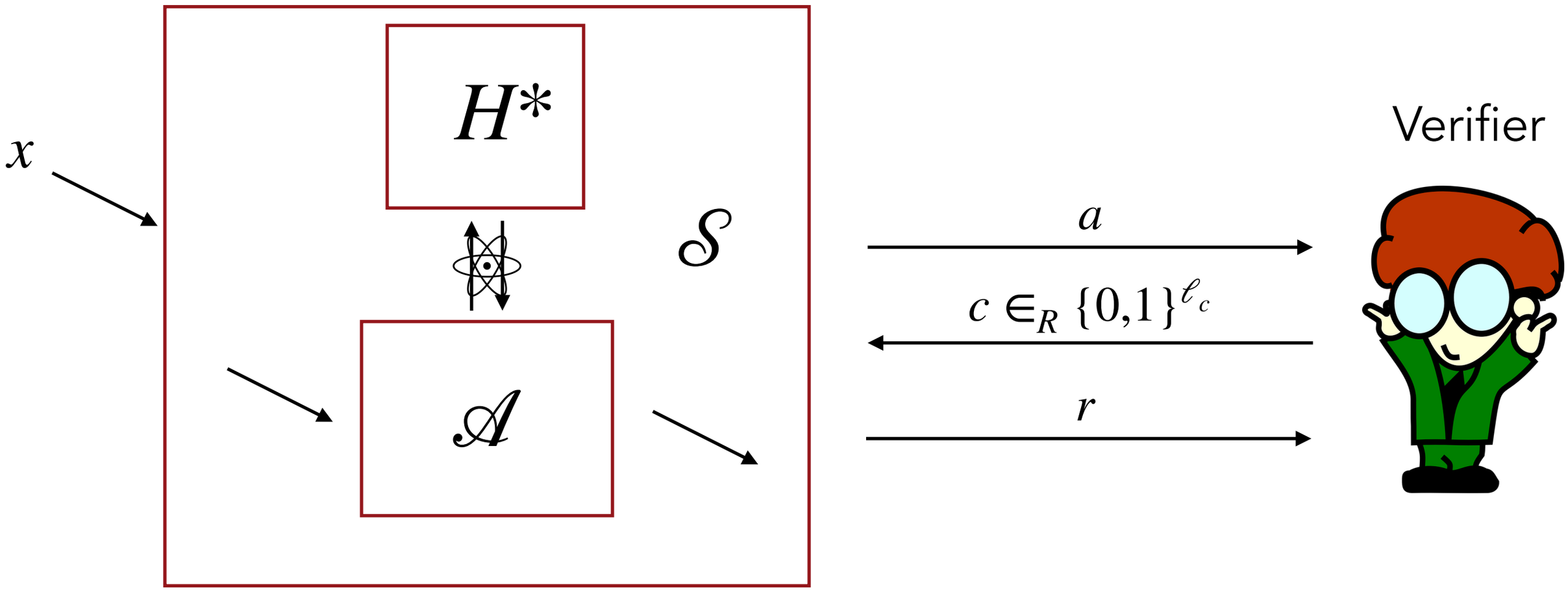

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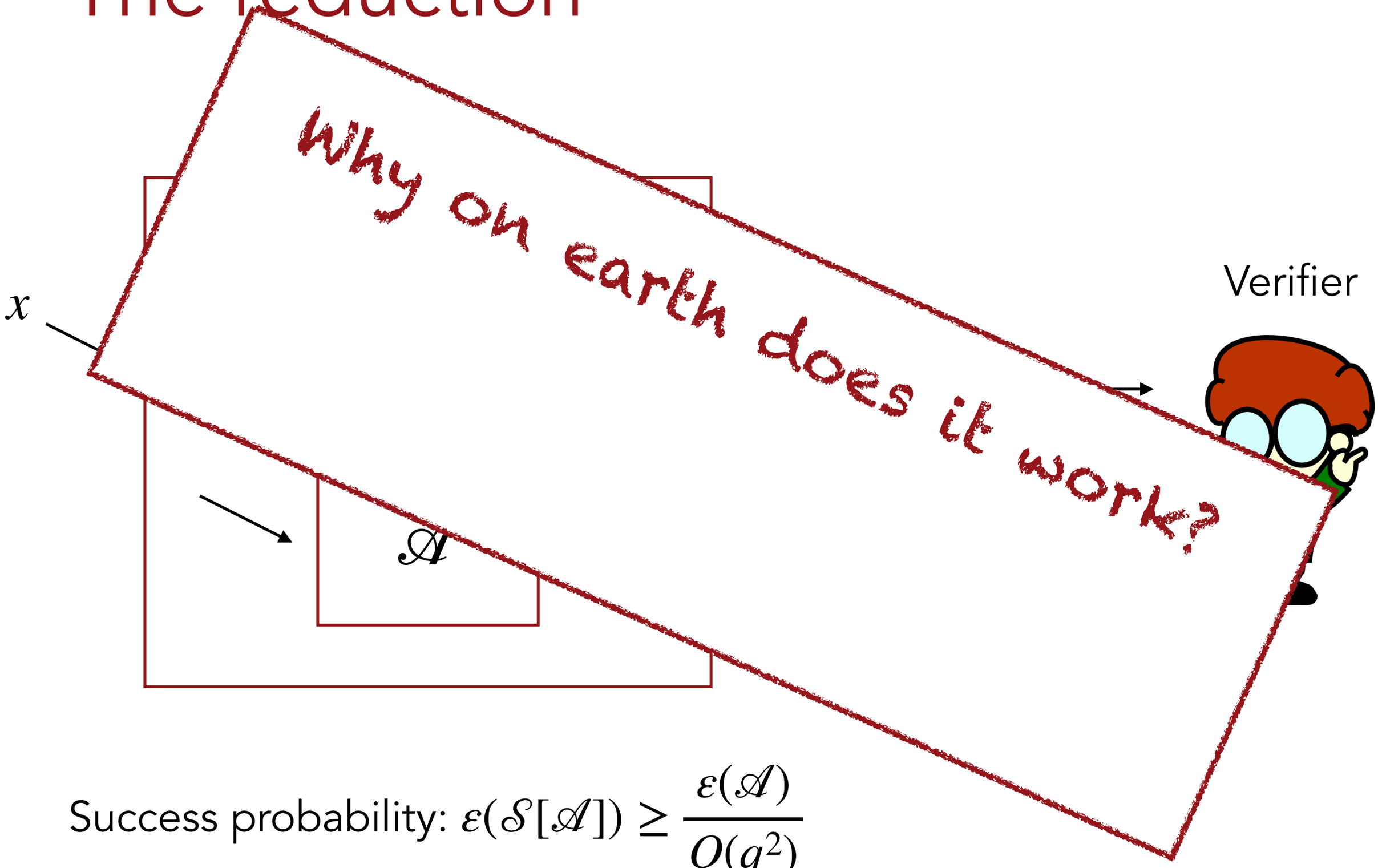

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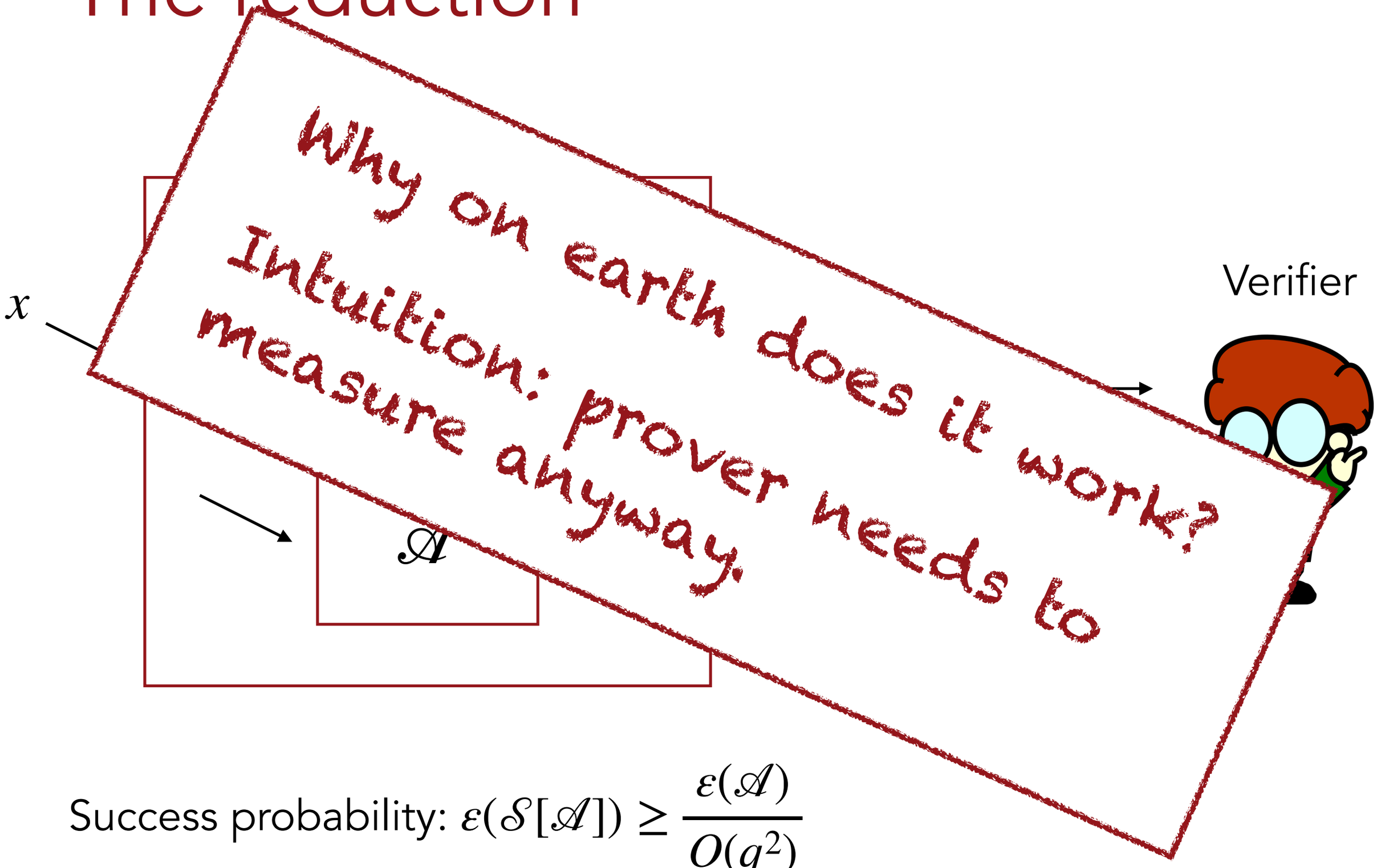

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2. Interpret RHS as algorithm

Technique

$$\mathscr{A}^{H^*}|\phi\rangle = \mathscr{A}^{H}|\phi\rangle + U_2 O_{H^*}|x_0\rangle\langle x_0|U_1|\phi\rangle - U_2 O_H|x_0\rangle\langle x_0|U_1|\phi\rangle \text{ (*)}$$

Plan: 1. Use (*) to test whether $\mathscr{A}^{H^*}$ outputs $|x_0\rangle|H^*(x_0)\rangle$

2. Interpret RHS as algorithm

$$\langle x_0|\langle H^*(x_0)|\mathscr{A}^{H^*}|\phi\rangle = \langle x_0|\langle H^*(x_0)|\mathscr{A}^{H}|\phi\rangle$$

$$+\langle x_0|\langle H^*(x_0)|U_2 O_{H^*}|x_0\rangle\langle x_0|U_1|\phi\rangle - \langle x_0|\langle H^*(x_0)|U_2 O_H|x_0\rangle\langle x_0|U_1|\phi\rangle$$

Technique

$$\mathscr{A}^{H}|\phi\rangle = \mathscr{A}^{H^*}|\phi\rangle + U_2 O_H |x_0\rangle\langle x_0| U_1 |\phi\rangle - U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \quad (*)$$

Plan: 1. Use (*) to test whether $\mathscr{A}^{H}$ outputs $|x_0\rangle|H(x_0)\rangle$

2. Interpret RHS as algorithm

$$\| \langle x_0| \langle H^*(x_0)| \mathscr{A}^{H^*} |\phi\rangle \|_2 \leq \| \langle x_0| \langle H^*(x_0)| \mathscr{A}^{H} |\phi\rangle \|_2$$

$$+ \| \langle x_0| \langle H^*(x_0)| U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2 + \| \langle x_0| \langle H^*(x_0)| U_2 O_{H} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2$$

Technique

$$\mathscr{A}^{H}|\phi\rangle = \mathscr{A}^{H^*}|\phi\rangle + U_2 O_H |x_0\rangle\langle x_0| U_1 |\phi\rangle - U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \quad (*)$$

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$$+ \| \langle x_0| \langle H^*(x_0)| U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2 + \| \langle x_0| \langle H^*(x_0)| U_2 O_{H} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2$$

Small even after summing over x_0

Technique

$$\mathscr{A}^{H}|\phi\rangle = \mathscr{A}^{H^*}|\phi\rangle + U_2 O_H |x_0\rangle\langle x_0| U_1 |\phi\rangle - U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \quad (*)$$

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$$+ \| \langle x_0| \langle H^*(x_0)| U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2 + \| \langle x_0| \langle H^*(x_0)| U_2 O_{H} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2$$

Measure query, outcome x_0, reprogram before answering

Technique

$$\mathscr{A}^{H}|\phi\rangle = \mathscr{A}^{H^*}|\phi\rangle + U_2 O_H |x_0\rangle\langle x_0| U_1 |\phi\rangle - U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \quad (*)$$

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$$\| \langle x_0| \langle H^*(x_0)| \mathscr{A}^{H^*} |\phi\rangle \|_2 \leq \| \langle x_0| \langle H^*(x_0)| \mathscr{A}^{H} |\phi\rangle \|_2$$

$$+ \| \langle x_0| \langle H^*(x_0)| U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2 + \| \langle x_0| \langle H^*(x_0)| U_2 O_{H} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2$$

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Technique

$$\mathcal{A}^{H}|\phi\rangle = \mathcal{A}^{H^*}|\phi\rangle + U_2 O_H |x_0\rangle\langle x_0| U_1 |\phi\rangle - U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \quad (*)$$

Plan: 1. Use (*) to test whether $\mathcal{A}^{H}$ outputs $|x_0\rangle|H(x_0)\rangle$

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$$+ \| \langle x_0| \langle H^*(x_0)| U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2 + \| \langle x_0| \langle H^*(x_0)| U_2 O_{H} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2$$

Square, Jensen's inequality ⇒RHS: success probability of reduction, reprogramming before/after the measured query at random

Technique

$$\mathcal{A}^{H}|\phi\rangle = \mathcal{A}^{H^*}|\phi\rangle + U_2 O_H |x_0\rangle\langle x_0| U_1 |\phi\rangle - U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \quad (*)$$

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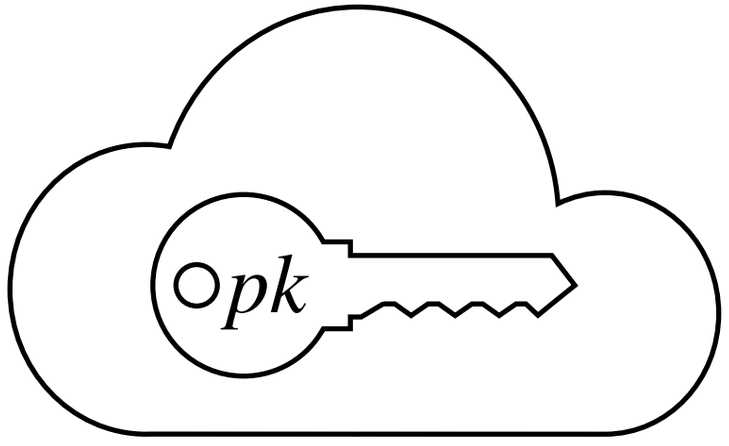
$$+ \| \langle x_0| \langle H^*(x_0)| U_2 O_{H^*} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2 + \| \langle x_0| \langle H^*(x_0)| U_2 O_{H} |x_0\rangle\langle x_0| U_1 |\phi\rangle \|_2$$

Square, Jensen's inequality ⇒RHS: success probability of reduction, reprogramming before/after the measured query at random

q **queries:** use (*) for each query. $O(q^2)$ loss from Jensen, interpretation as expectation value

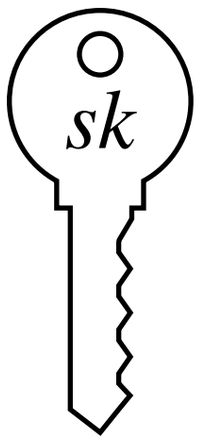
3. Application: Digital Signatures

Identification scheme



∃*sk* for *pk*!

Prove it!



Prover

a →

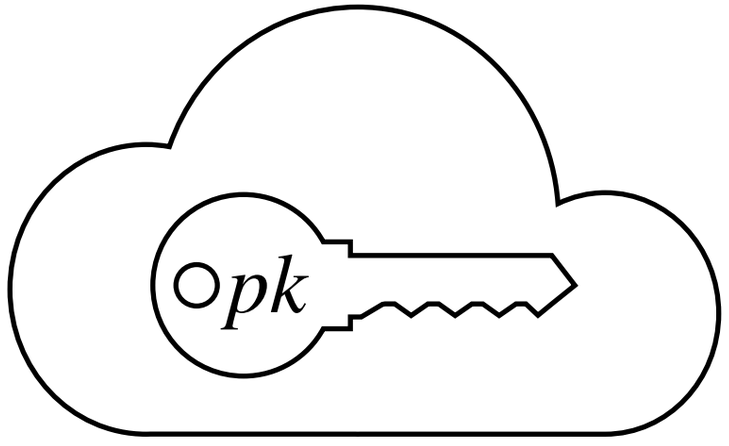
← $c \in_R \mathscr{C}$

r →

Verifier

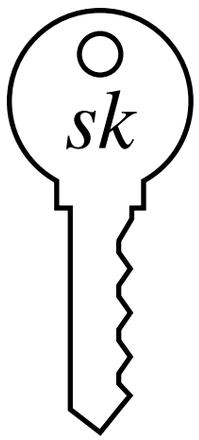
Now I believe
∃*sk* for *pk*…

Identification scheme



∃sk for pk!

Prove it!



Prover

a →

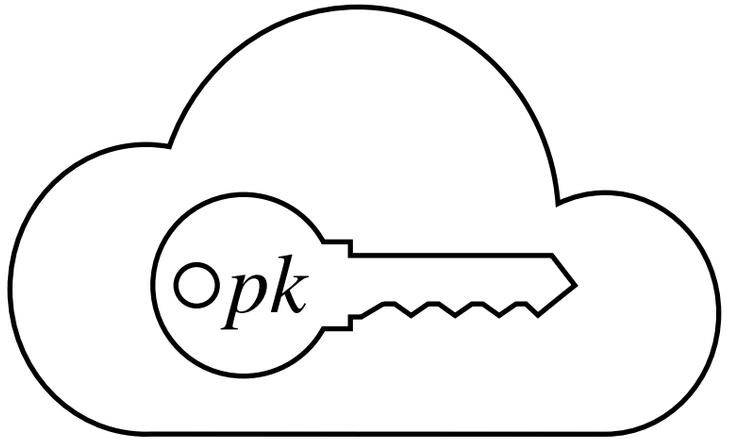
← $c \in_R \mathcal{C}$

r →

Verifier

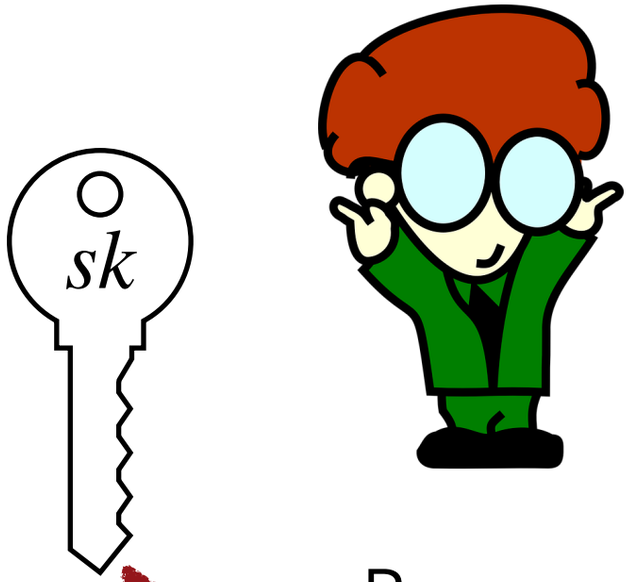
Now I believe that
Prover has sk for pk…

Identification scheme



∃*sk* for *pk*!

Prove it!



Prover

a

$c \in_R \mathcal{C}$

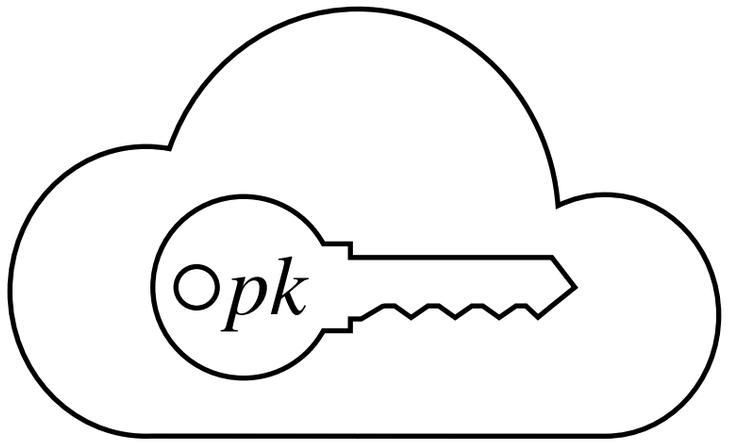
r

Verifier

Still private!

Now I believe that Prover has *sk* for *pk*…

Identification scheme



∃sk for pk!

Prove it!

An Identification scheme is a zero-knowledge proof of knowledge of a private key.

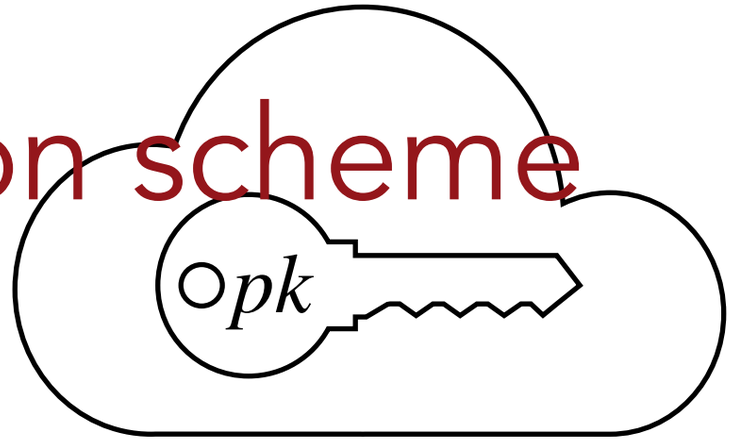
Still private!

Prover

Verifier

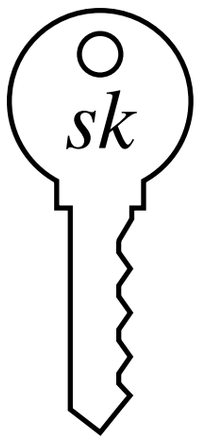
Now I believe that Prover has sk for pk…

Noninteractive Identification scheme



∃*sk* for *pk*!

Prove it!



Prover

a →

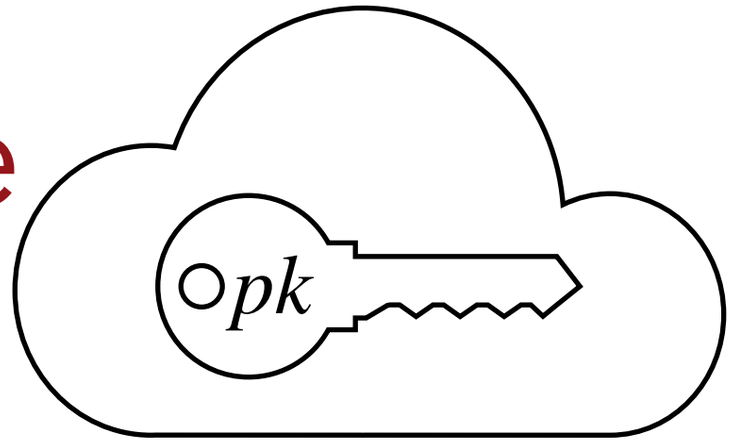
$c = H(a)$ →

r →

Verifier

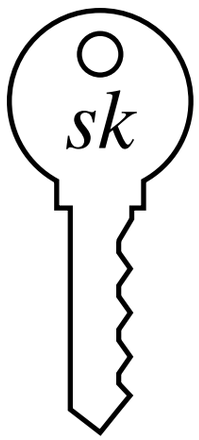
Now I believe that
Prover has *sk* for *pk*…

Digital signature scheme



$\exists sk$ for pk!

Prove it!



Prover

a →

$c = H(a \| m)$ →

r →

Verifier

Now I believe that Prover has used sk to sign m

Fiat Shamir signatures

Several NIST post-quantum candidates use Fiat Shamir:

Fiat Shamir signatures

Several NIST post-quantum candidates use Fiat Shamir:

- Picnic
- Dilithium
- MQDSS
- QTesla

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Our result ⇒ QROM security

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Improved efficiency!

Fiat Shamir signatures

Several NIST post-quantum candidates use Fiat Shamir:

- Picnic
- Dilithium
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Our result ⇒ QROM security

Improved efficiency!

Further applications

Remove almost all interaction from Mahadev's verification for BQP (Alagic, Childs, Hung '19)

Summary

The Fiat Shamir transformation is secure in the quantum random oracle model.

This fact has nice applications, in particular for post-quantum secure digital signature schemes.

Open problem: ∃ quantum forking lemma?

Thanks!