## On Attacking Hash functions in Cryptographic schemes

Workshop "Quantum cryptanalysis of post-quantum cryptography" Simons institute for the Theory of Computing

#### Christian Majenz



**USOFT** Research Center for Quantum Software









- Commitments
- Noninteractive zero knowledge
- ...



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#### Outline

1.Intro: Hash functions

- i. Basics, security
- ii.The (quantum) random oracle model
- iii.Domain extension
- 2.Points of attack
- 3.Hash-function-based generic transformations: Fiat-Shamir and Fujisaki-Okamoto
- 4.Attacks and attack approaches against Fiat-Shamir and Fujisaki-Okamoto

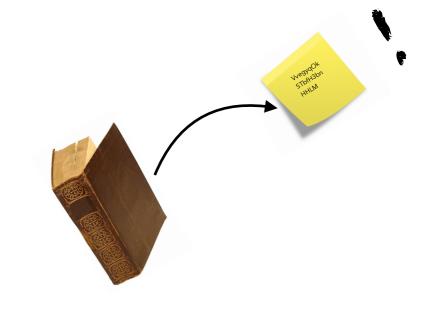
# Intro: Hash functions



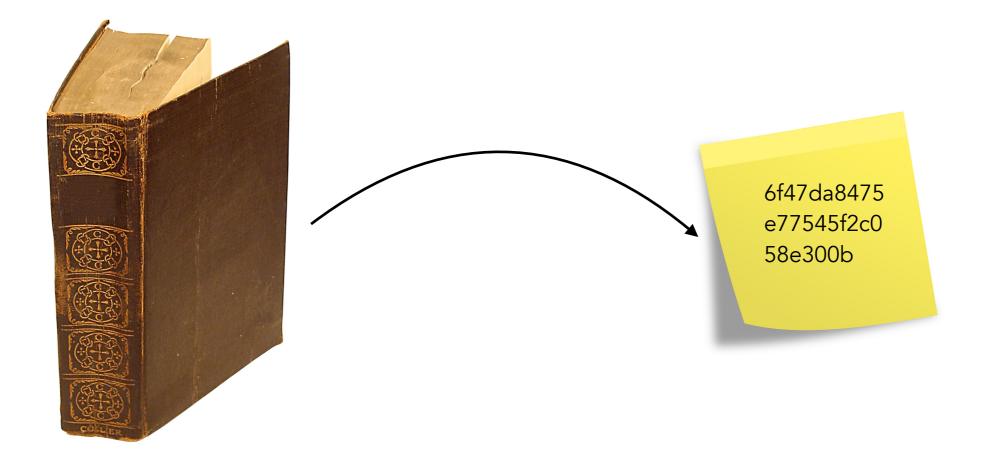


### What is a hash function?



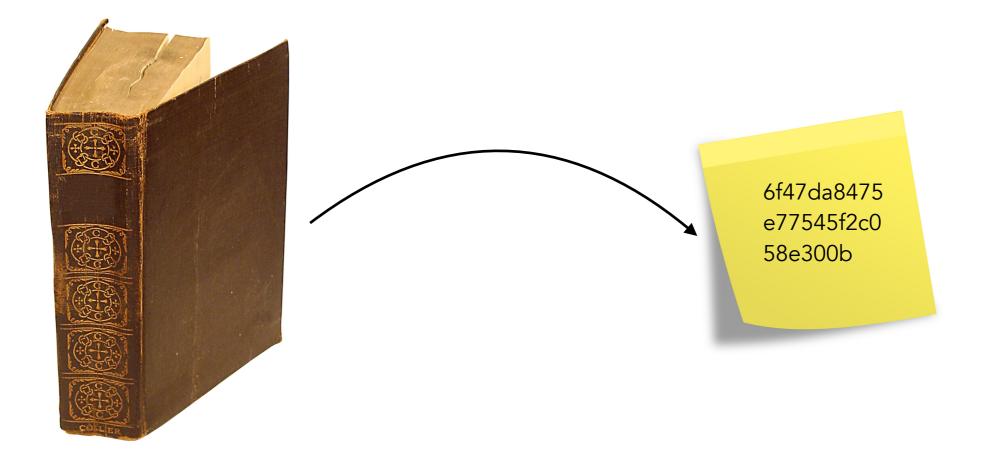


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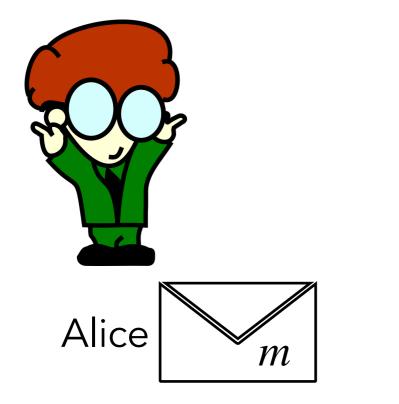
⇒(Quantum) Random Oracle Model





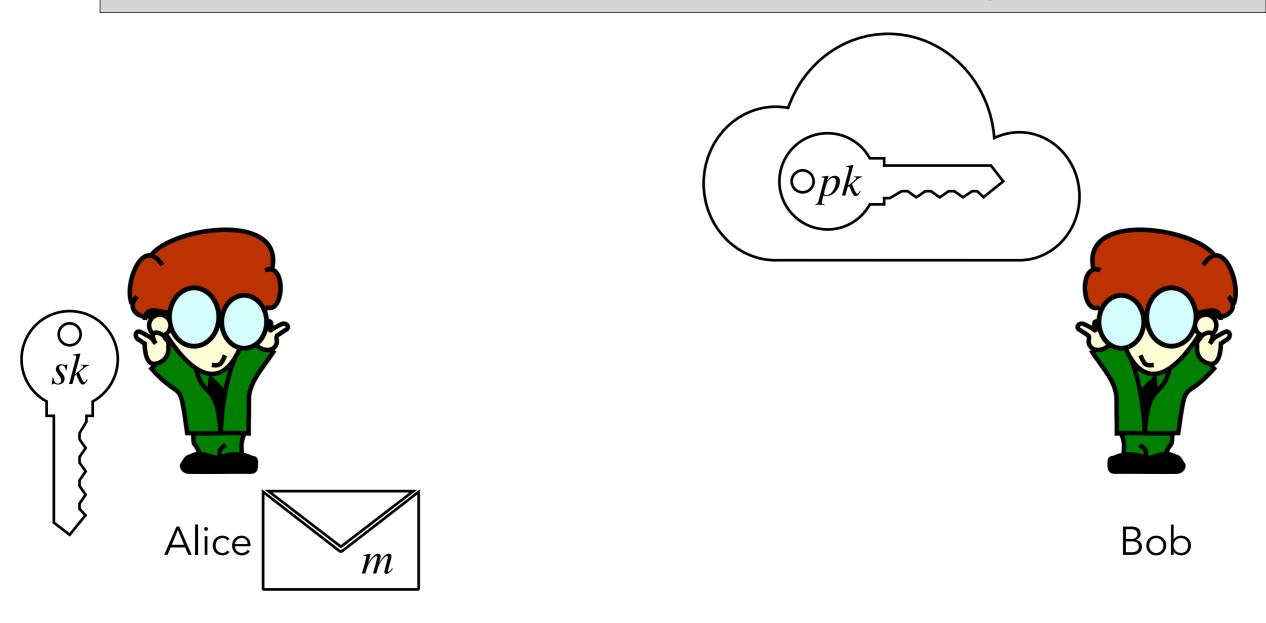
Alice

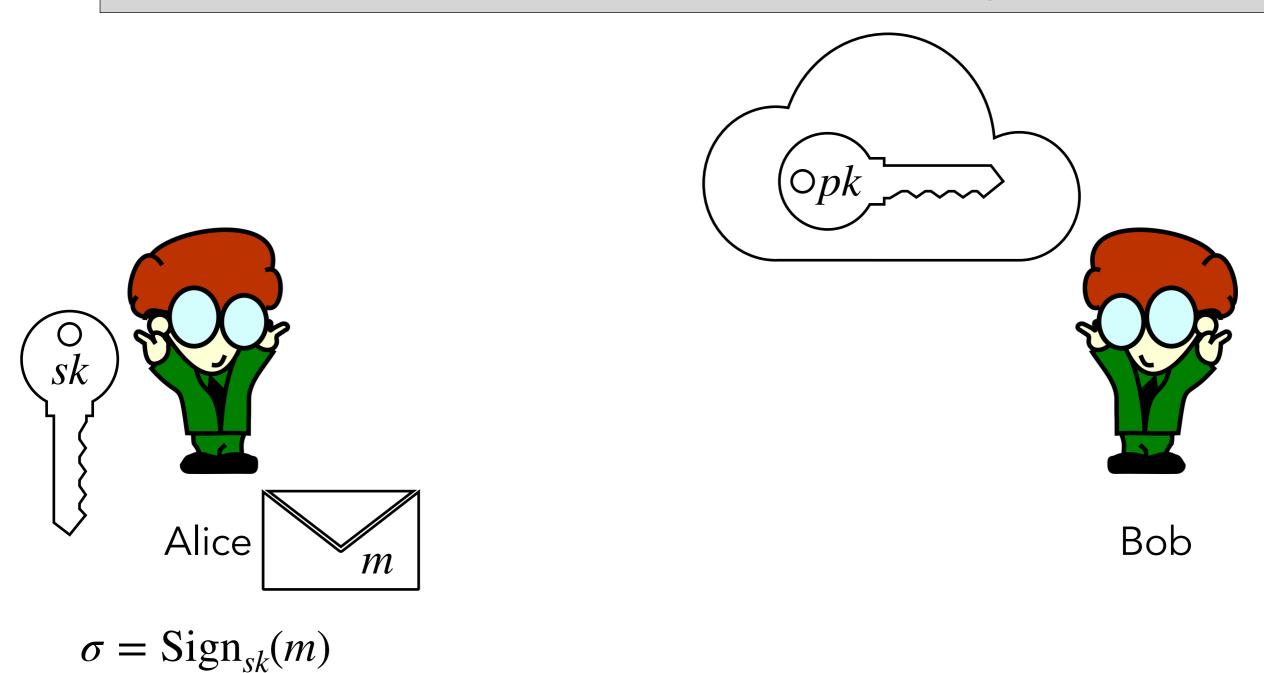
Bob

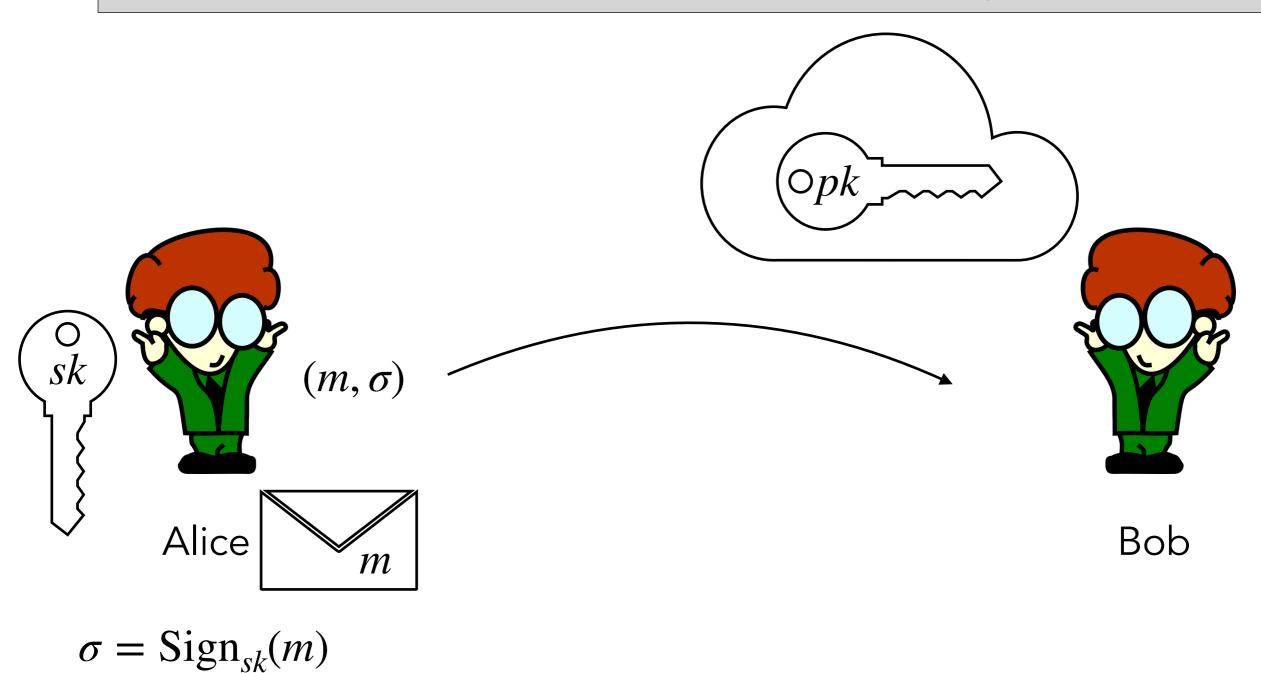


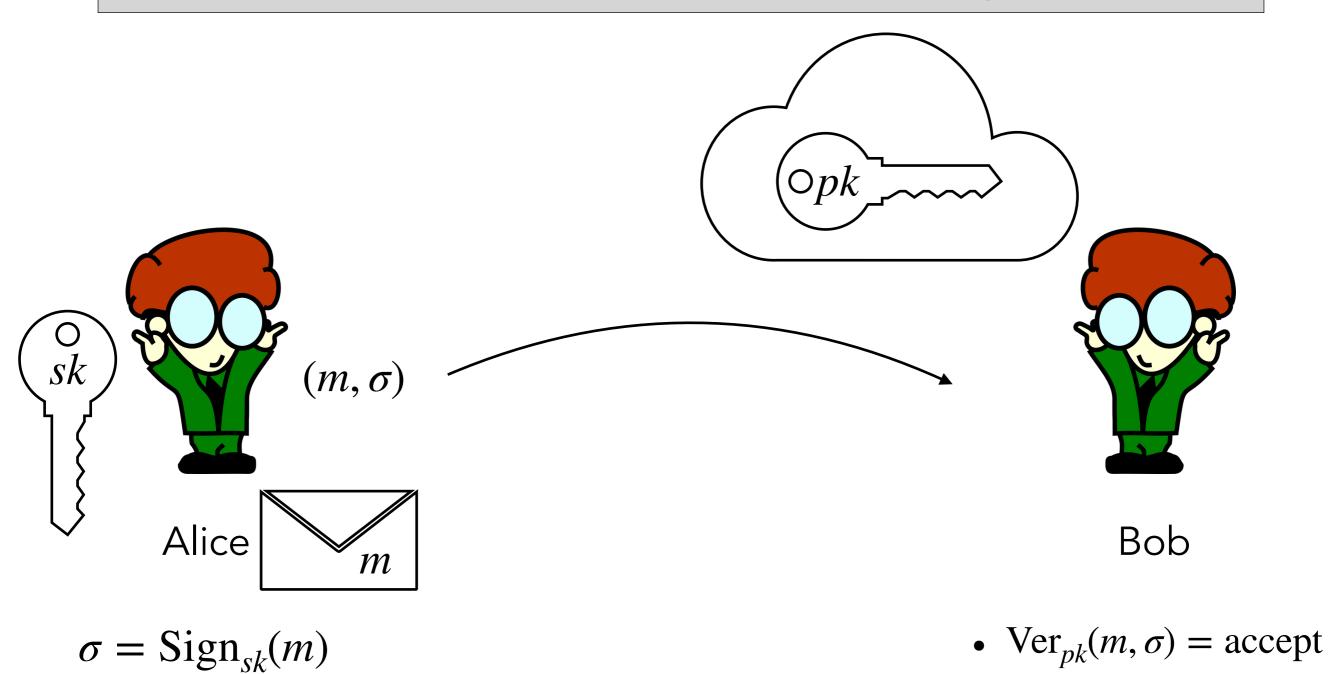


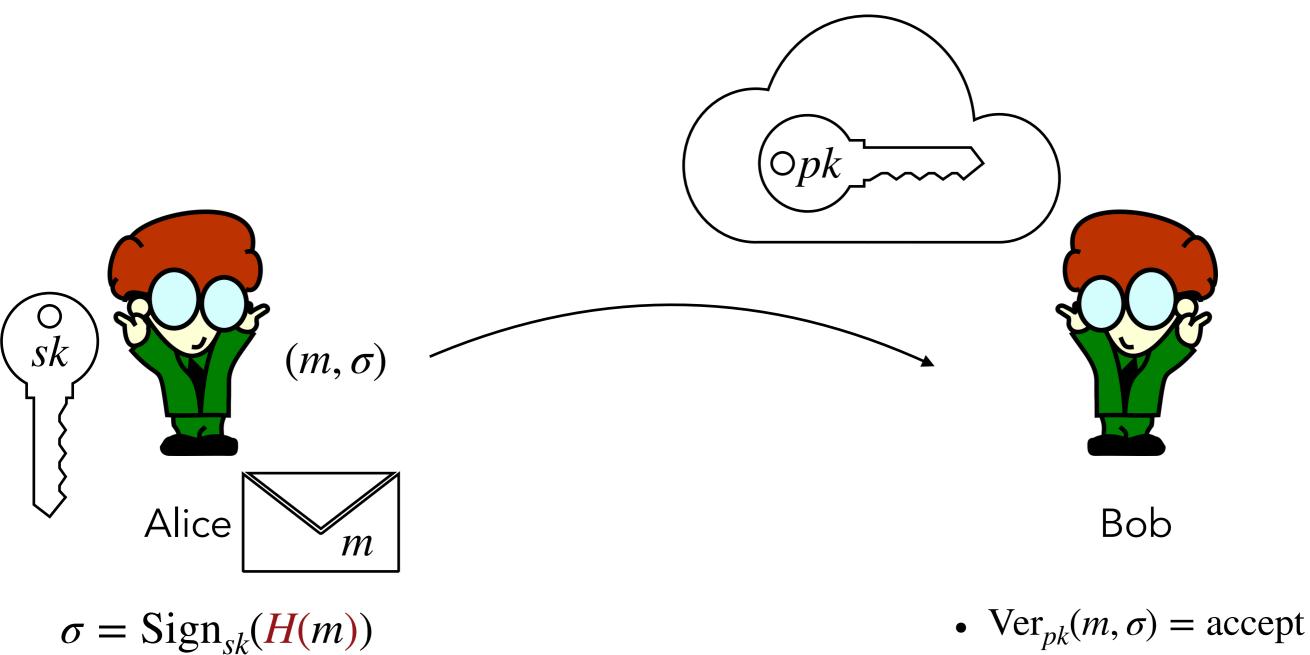
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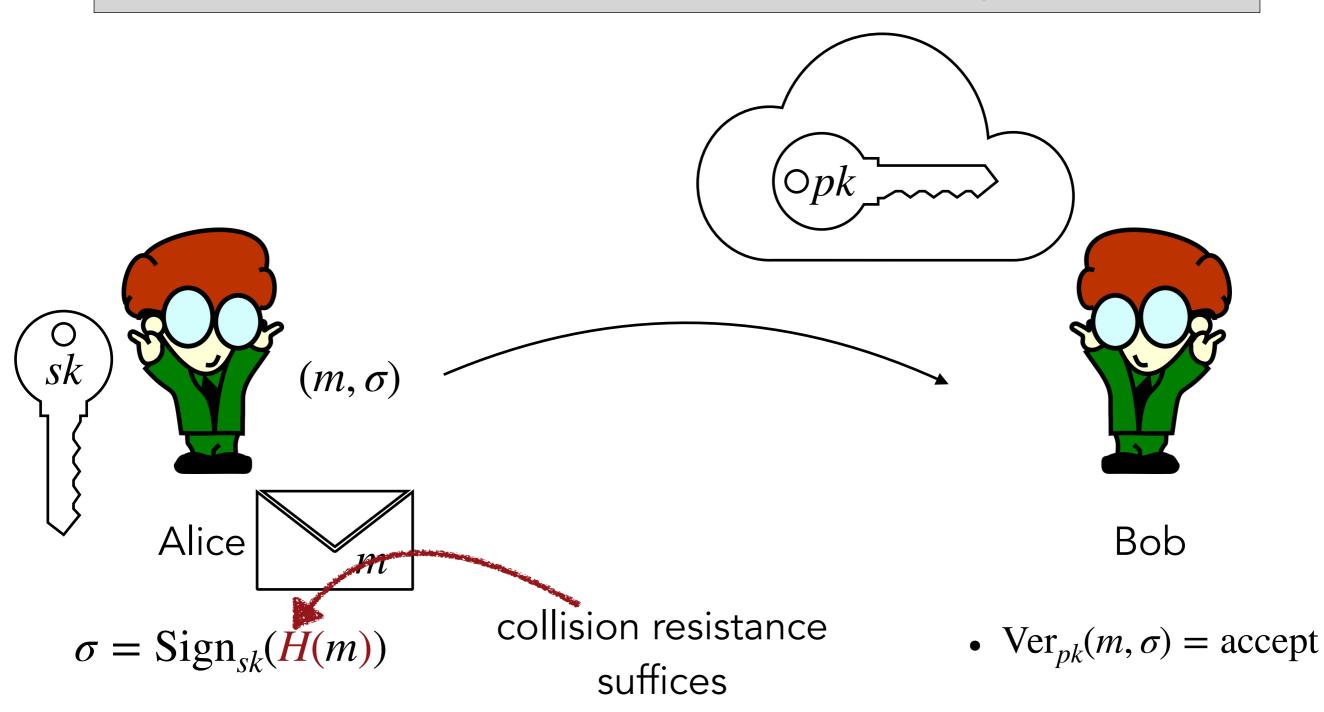












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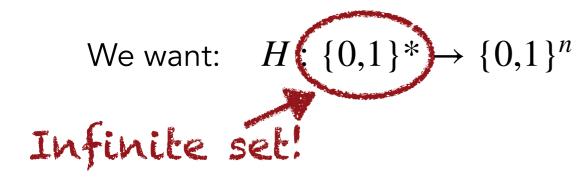
Allows public oracle access to  $|x\rangle |y\rangle \mapsto |x\rangle |y \oplus H(x)\rangle$ 

+ Has enabled security proofs for more efficient cryptographic schemes

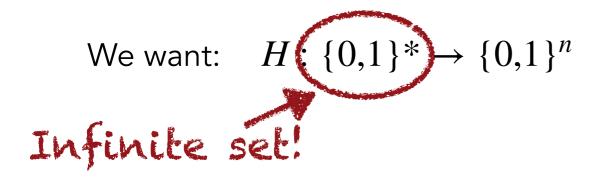
- It's not the real world!

We want:  $H: \{0,1\}^* \to \{0,1\}^n$ 

#### **Domain extension**

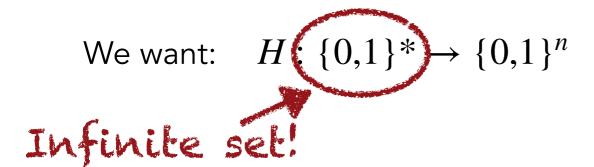


#### **Domain extension**



Easier:  $f: \{0,1\}^k \to \{0,1\}^{\ell}$ 

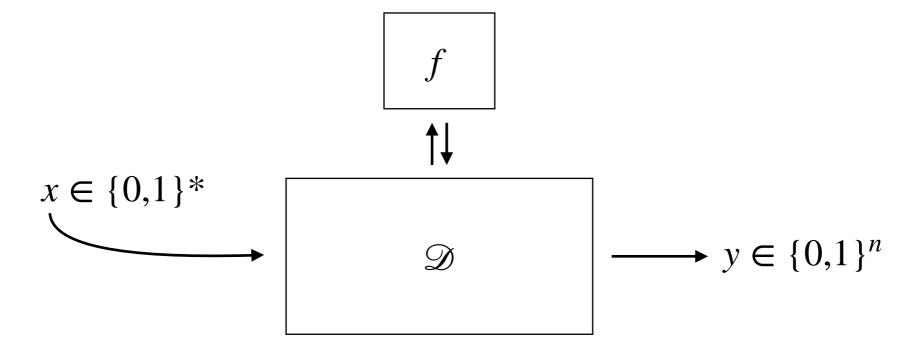
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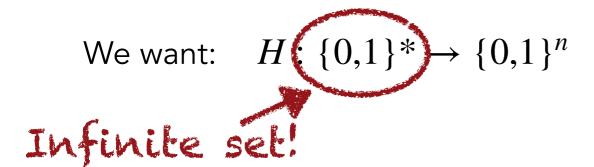
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Domain extension scheme  $\mathcal{D}$ : compute y = H(x) by



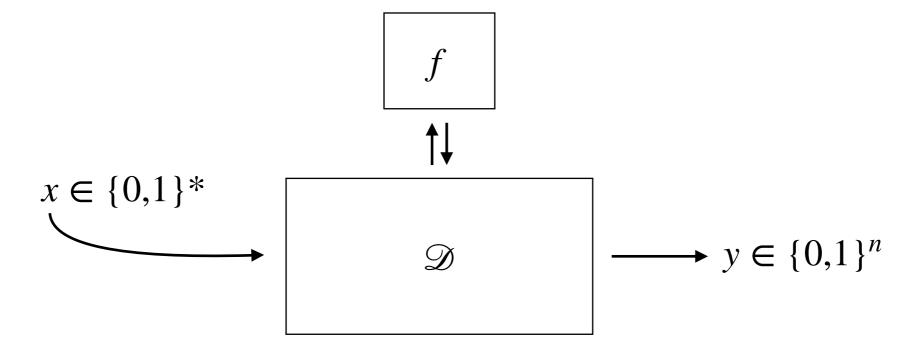
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SHA-1 SHA-2, SHA-3 work like this.

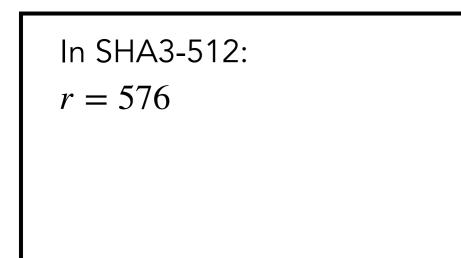
#### Example: the sponge construction

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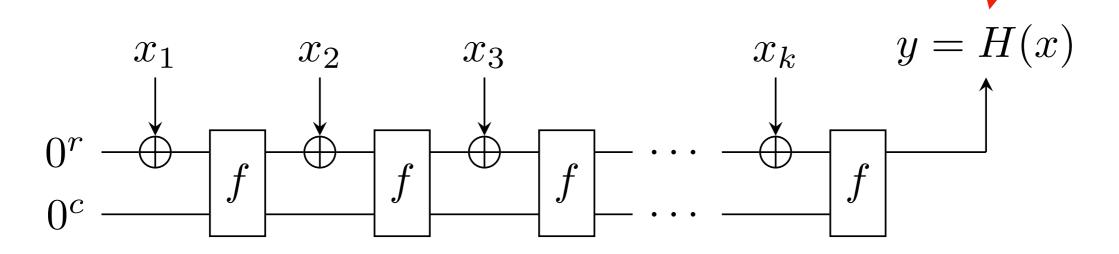
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A particular domain extension scheme used e.g. in SHA-3

H: split input x into chunks  $x_1, \ldots, x_k$  of r bits each and do



In SHA3-512:  

$$r = 576$$
  
 $c = 1024$ 



output

Digital signature schemes:

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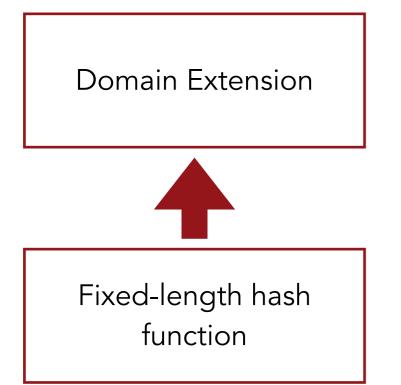
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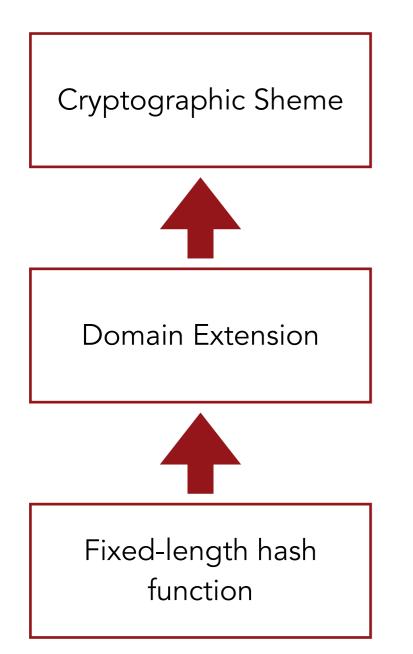
Key encapsulation schemes

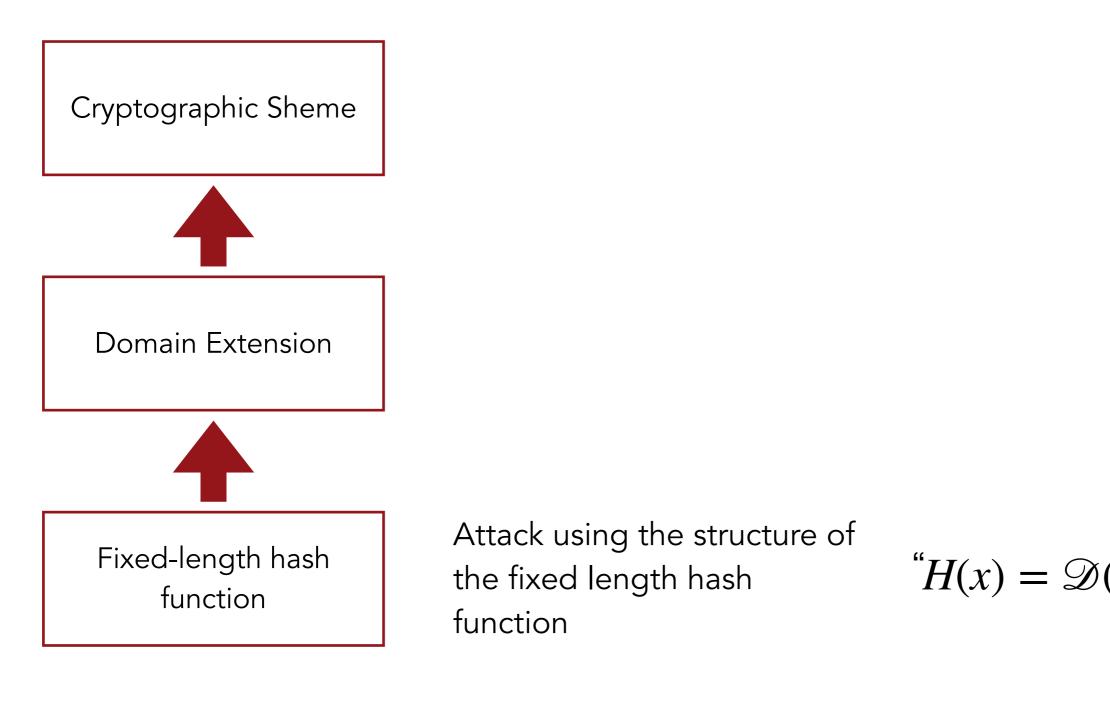
- All: hash-based key derivation
- Some: Fujisaki-Okamoto Transformation

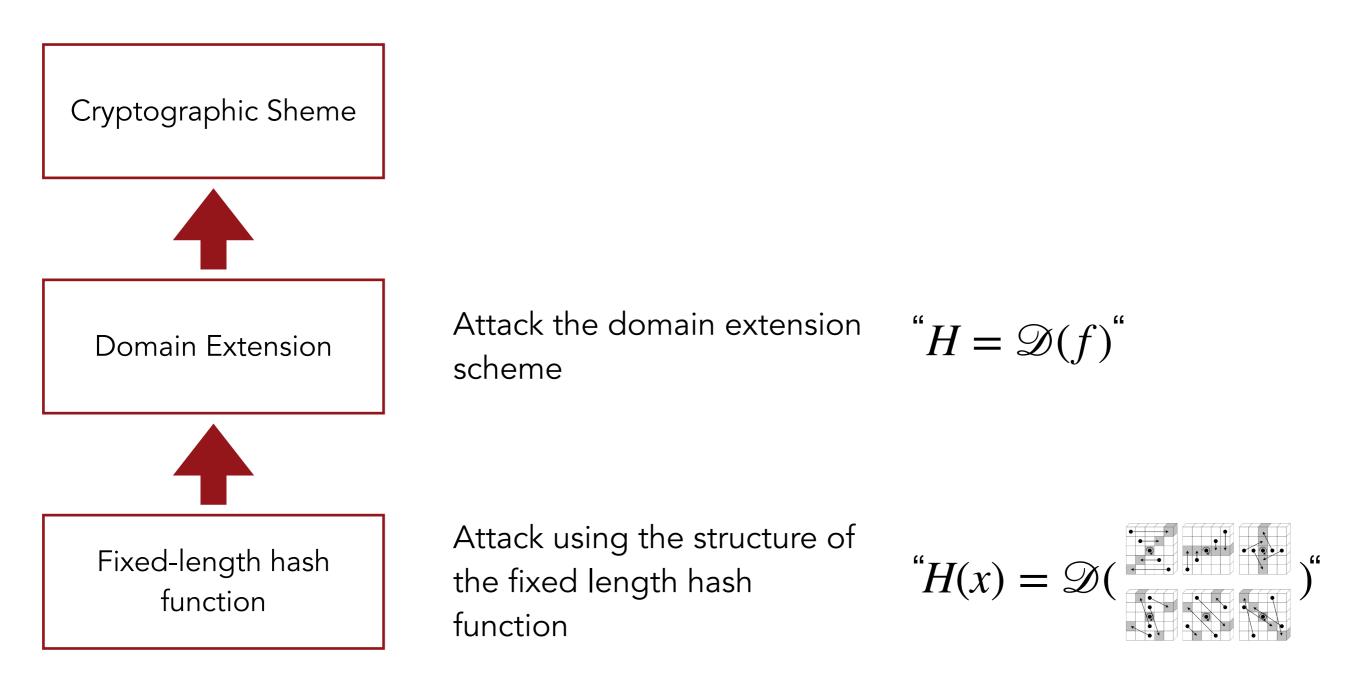
# Points of attack

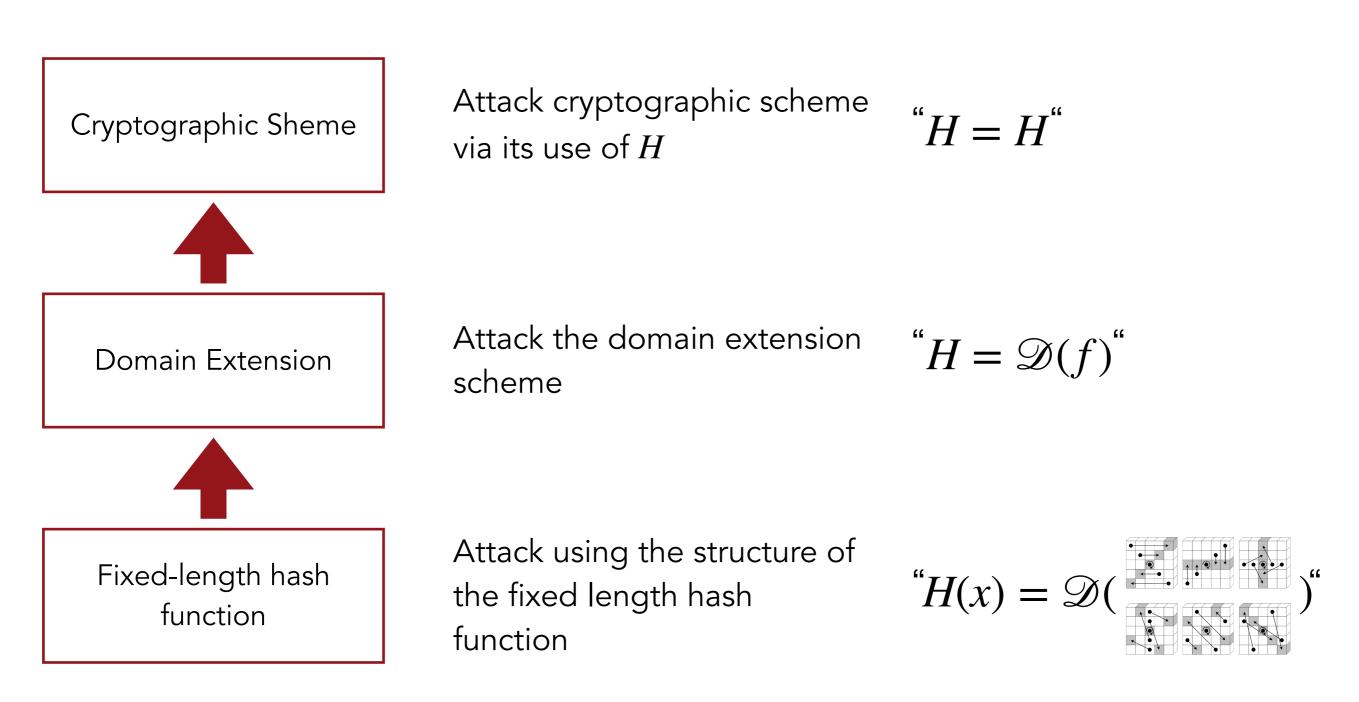
Fixed-length hash function











Attack using the structure of the fixed length hash function

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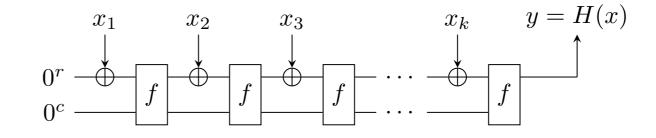
# Finding Hash Collisions with Quantum Computers by Using Differential Trails with Smaller Probability than Birthday Bound

Akinori Hosoyamada<sup>1,2</sup> and Yu Sasaki<sup>1</sup>

 <sup>1</sup> NTT Secure Platform Laboratories, Tokyo, Japan, {akinori.hosoyamada.bh,yu.sasaki.sk}@hco.ntt.co.jp
 <sup>2</sup> Nagoya University, Nagoya, Japan, hosoyamada.akinori@nagoya-u.jp

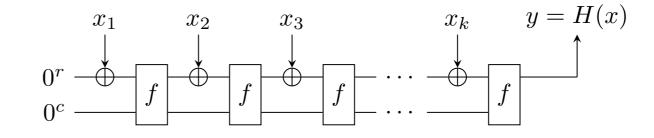
Attack the domain extension scheme

Attack the domain extension scheme



**Theorem 16.** Let  $\mathbf{S}_{c,r,\mathbf{f},pad,n}(m)$  be a sponge construction with arbitrary block function  $\mathbf{f}$ . There exists a quantum algorithm COLL-RO making at most  $q_{\mathbf{f}}$  quantum queries to  $\mathbf{f}$  and  $q_{\mathcal{H}}$  quantum queries to a random oracle  $\mathcal{H}$ . COLL-RO outputs colliding messages  $m \neq \hat{m}$  such that  $\mathbf{S}_{c,r,\mathbf{f},pad,n}(m) = \mathbf{S}_{c,r,\mathbf{f},pad,n}(\hat{m})$  with probability at least 1/8, where  $q_{\mathbf{f}} := 2k_{\text{Amb}} \cdot \min\{\frac{c+6+2r}{r}2^{c/3}, \frac{2n+6+3r}{r}2^{n/3}\}$ , and  $q_{\mathcal{H}} := 2k_{\text{Amb}} \cdot \min\{2^{c/3}, 2^{n/3}\} + 2$ , where  $k_{\text{Amb}}$  is the constant from Theorem 14 and pad is any padding function which appends at most 2r bits.

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Finds collision for sponge by finding collision of f

Attack cryptographic scheme via its use of H

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# Remainder of this talk: 2 Examples

Fiat-Shamir and Fujisaki-Okamoto



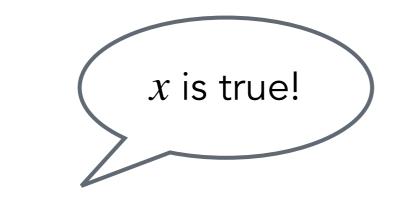
#### Prover



Prover



Verifier





Prover

Verifier



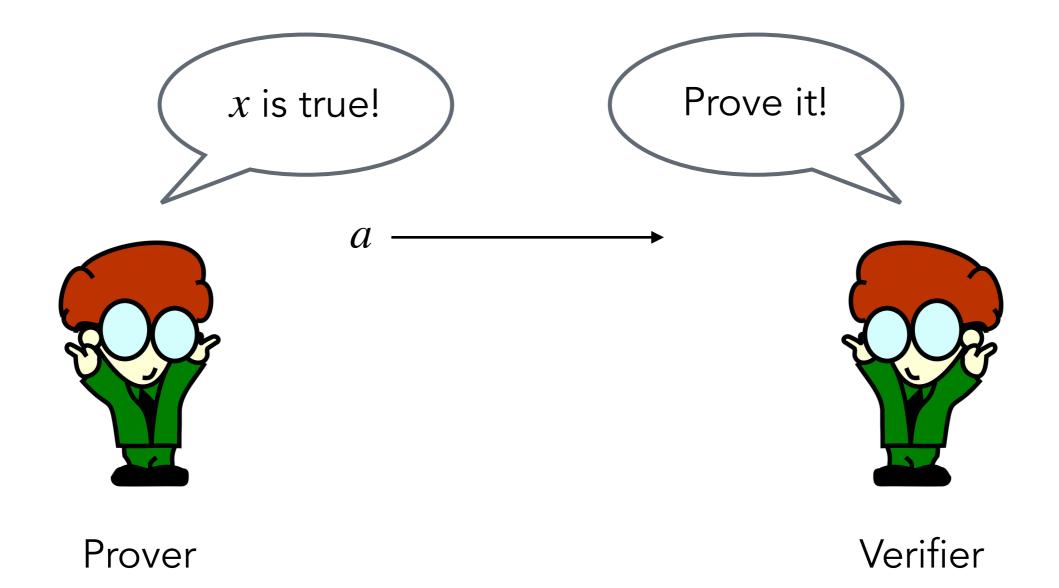


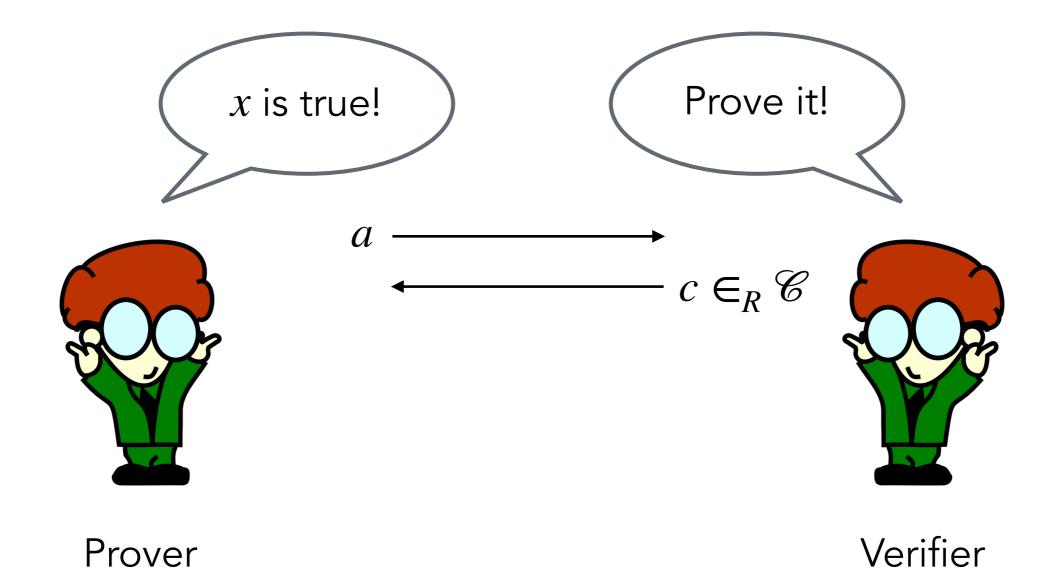


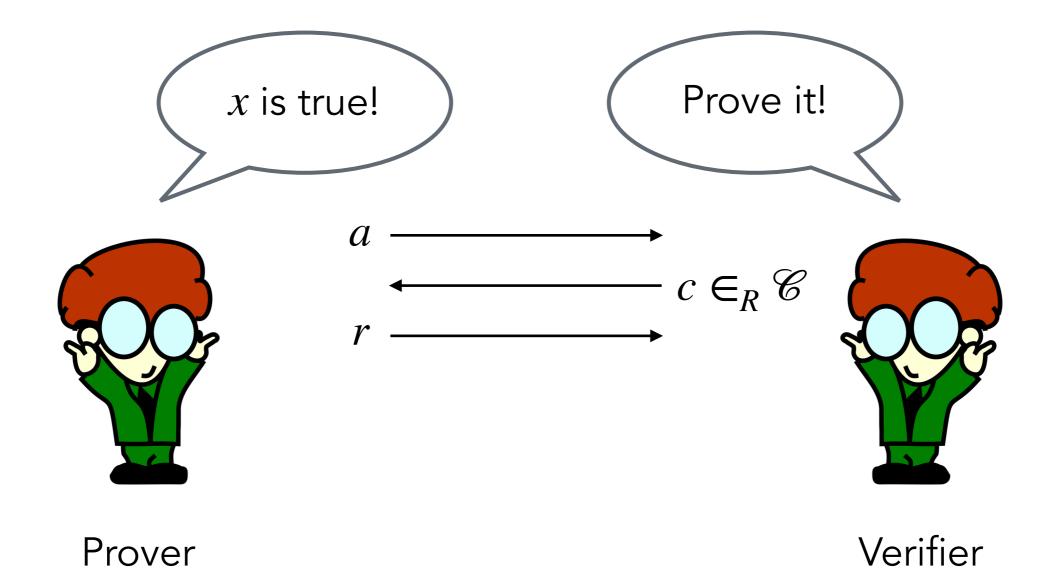


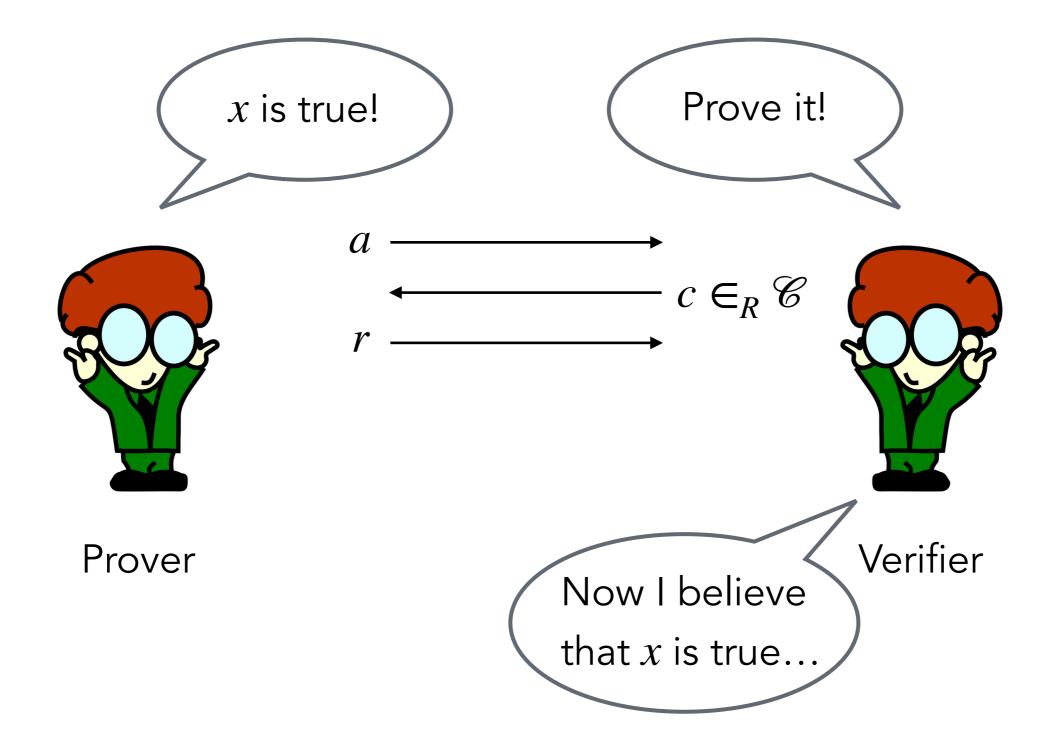
Prover

Verifier

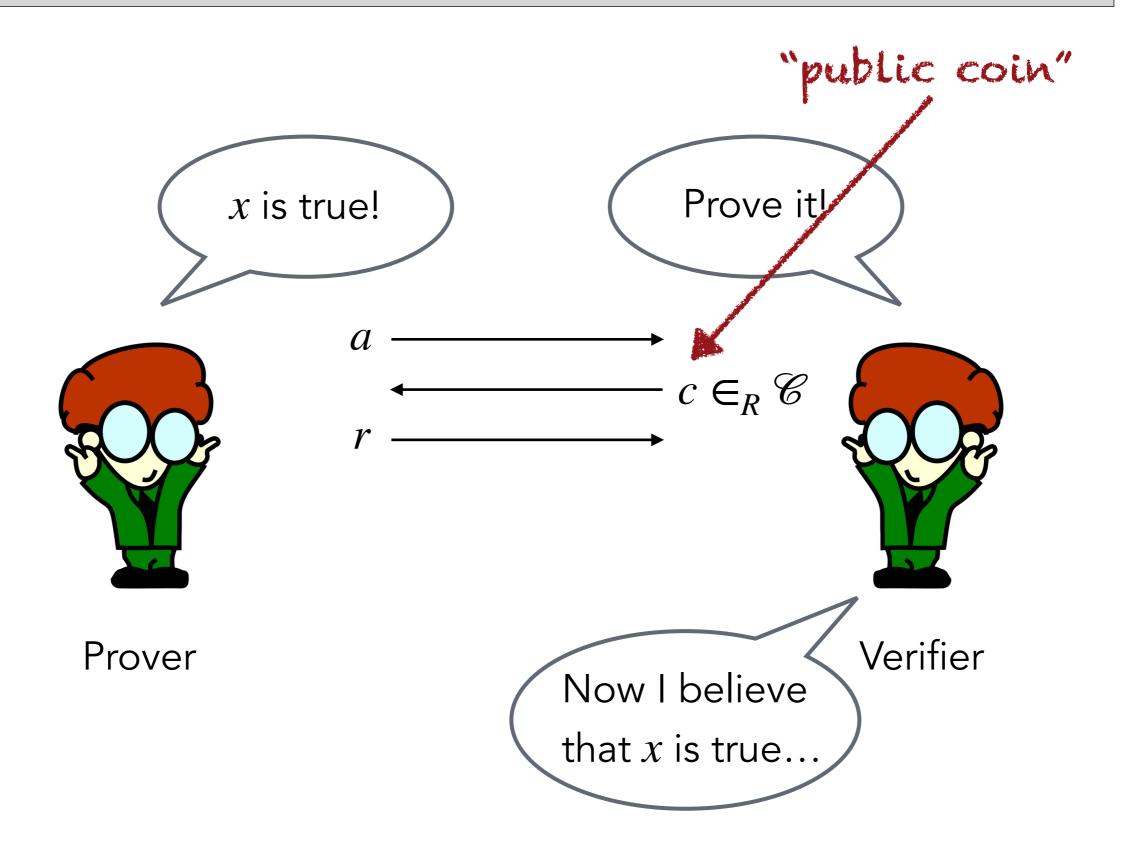




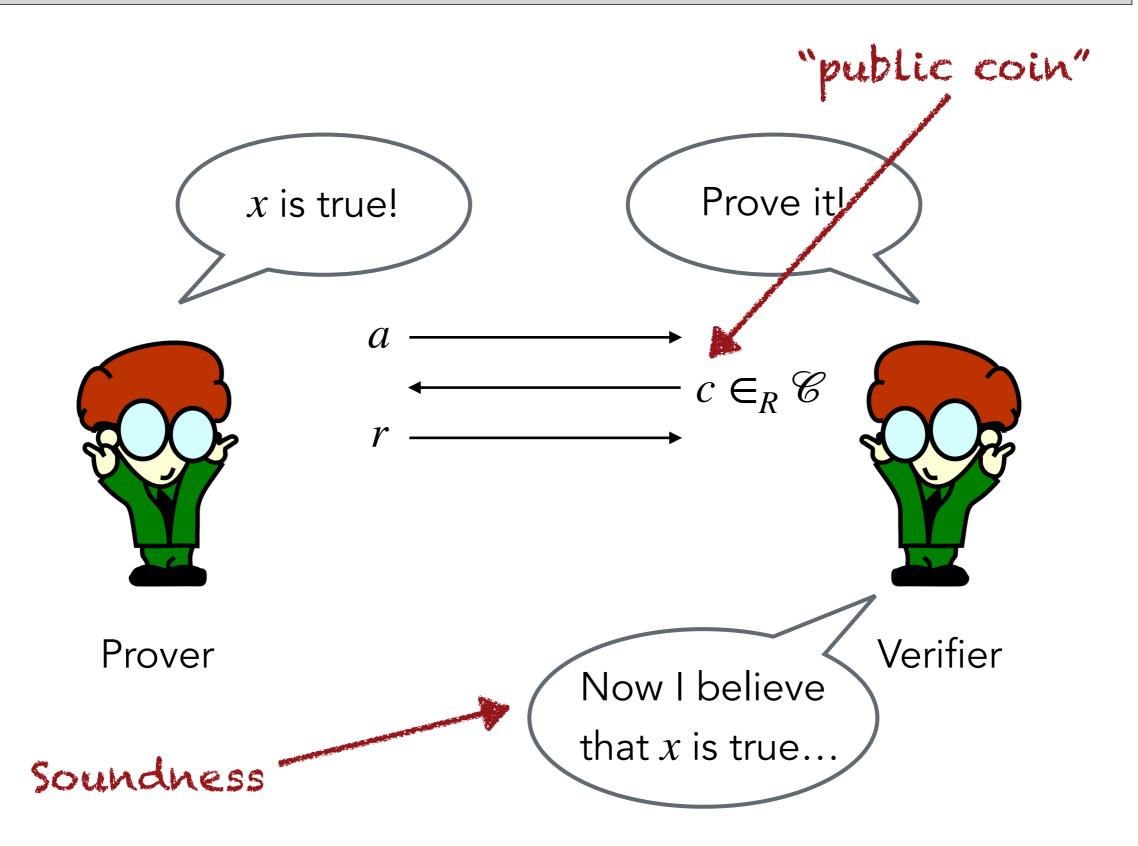




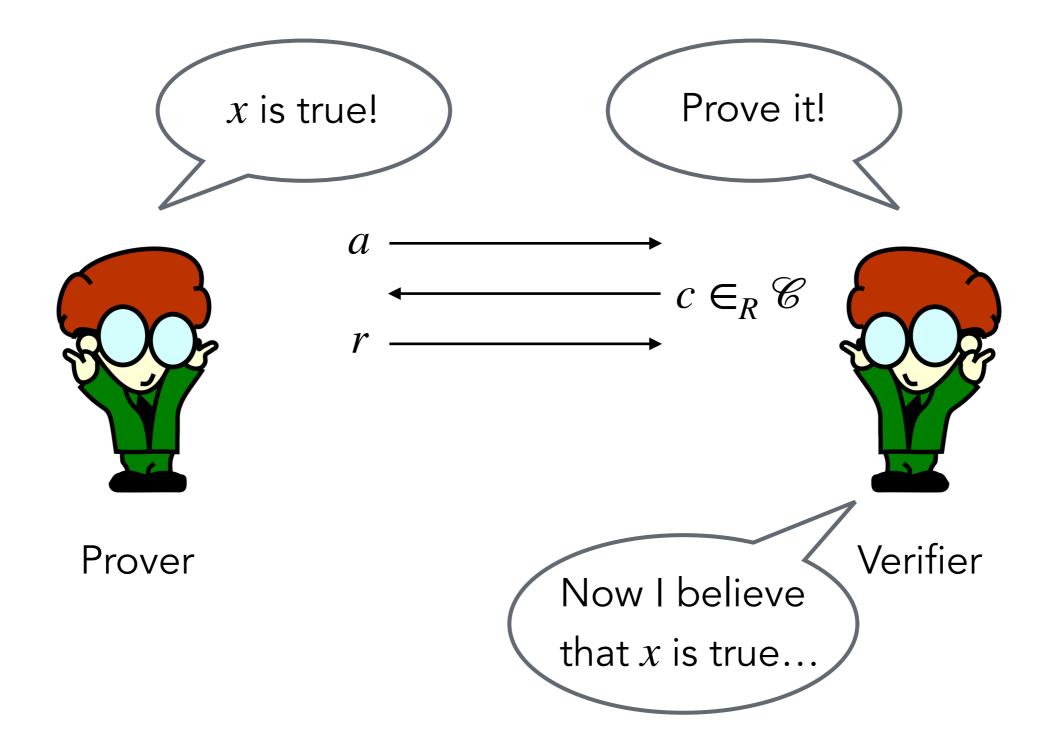
#### Sigma-protocols



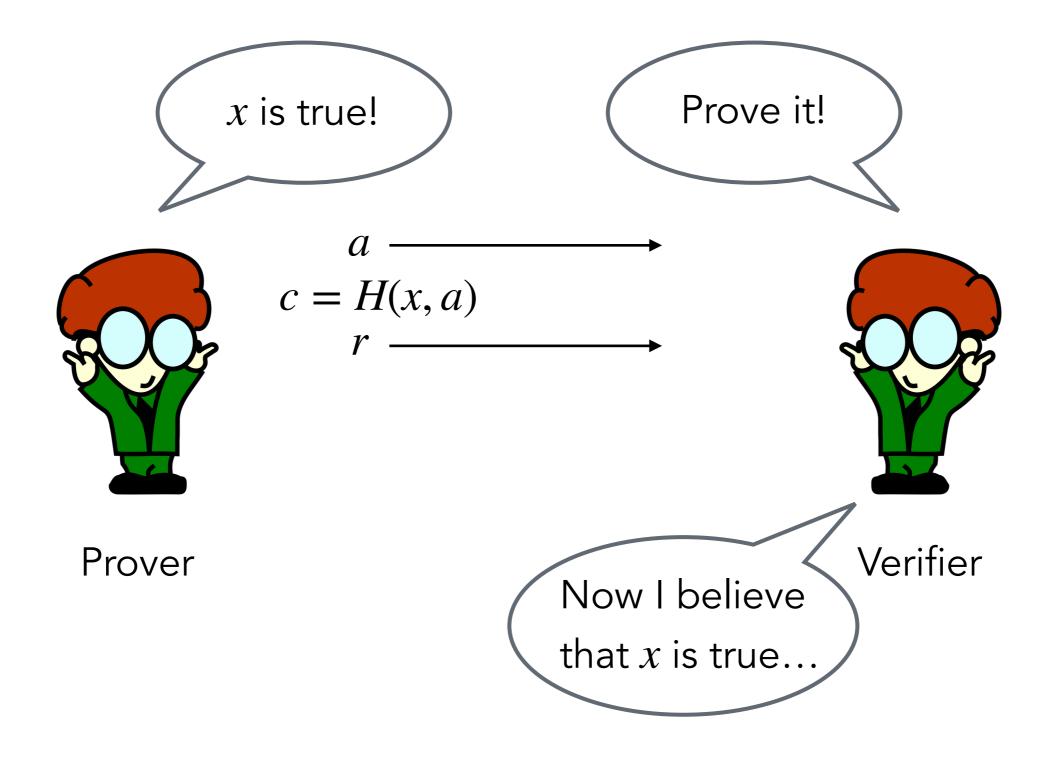
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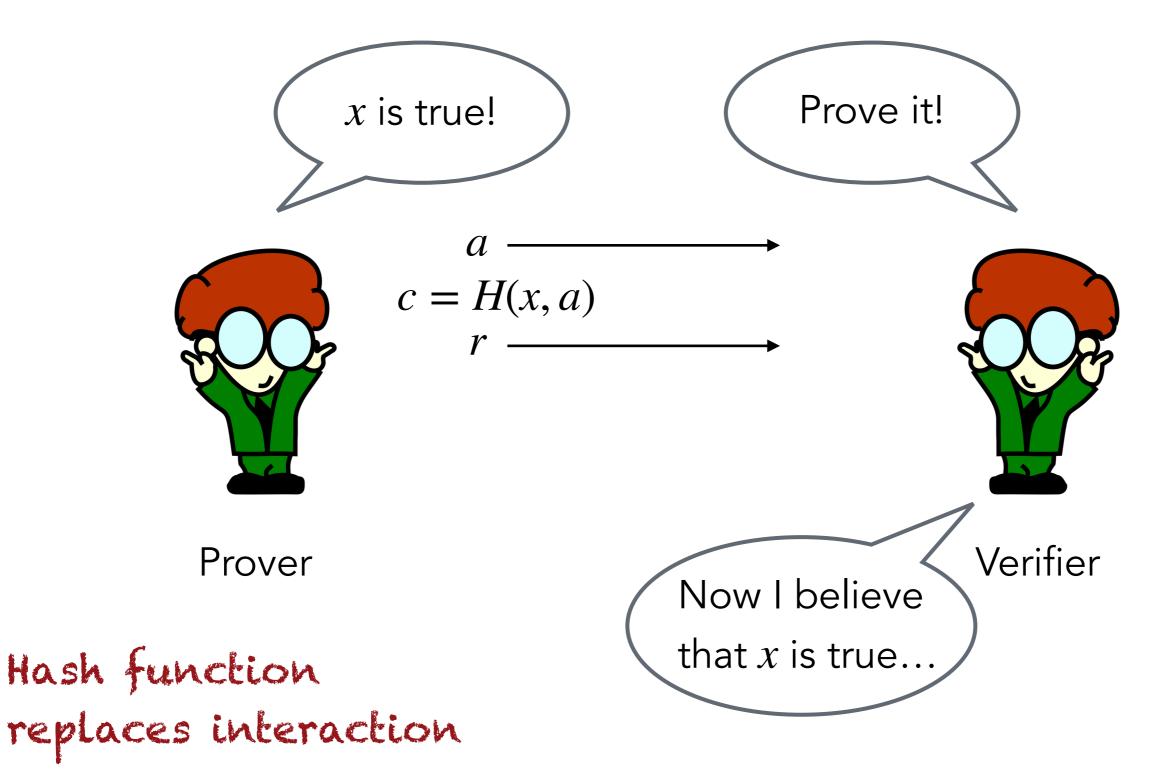
#### **Fiat-Shamir transformation**



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# Fiat-Shamir signature scheme O x*m* comes Prove it! from me $\bigcirc$ W a c = H(x, a, m)r Verifier Prover Now I believe that you sent m...

#### **Fujisaki-Okamoto transformation**

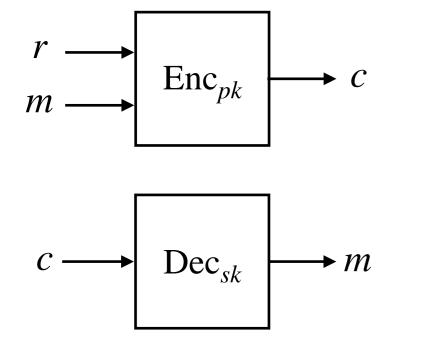
Upgrades weak security to chosen-ciphertext security for key encapsulation

"Derandomize, Hash&reincrypt"

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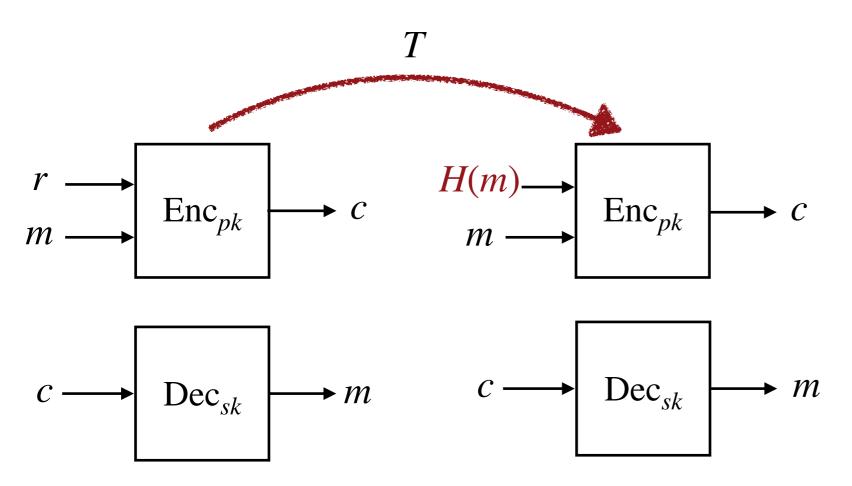
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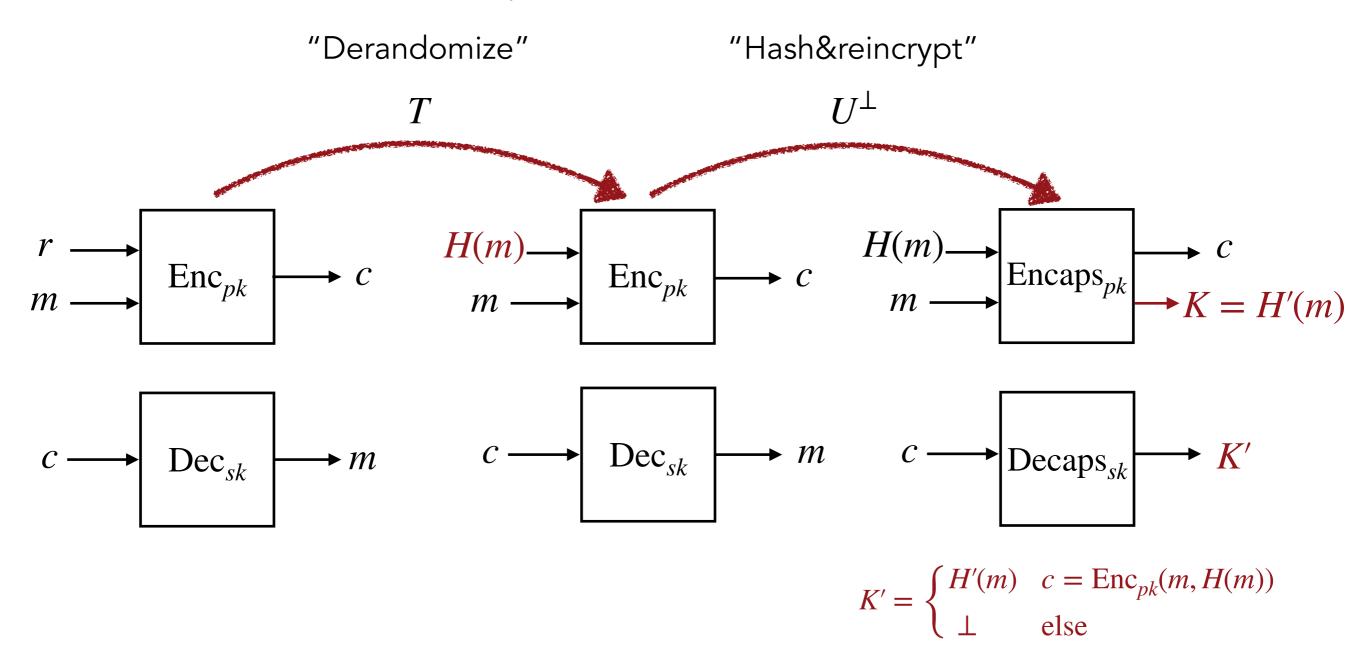
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Attacks and attack approaches

#### Fiat-Shamir transformation in the QROM

**Theorem** (Don, Fehr, M, Schaffner '19):

An dishonest prover making q quantum queries to the random

oracle can prove a wrong statement in the Fiat-Shamir

Transformation  $\mathsf{FS}(\Sigma)$  of a sigma protocol  $\Sigma$  with probability at most

$$\varepsilon_{\mathsf{FS}(\Sigma)}(q) \le (2q+1)^2 \varepsilon_{\Sigma'}$$

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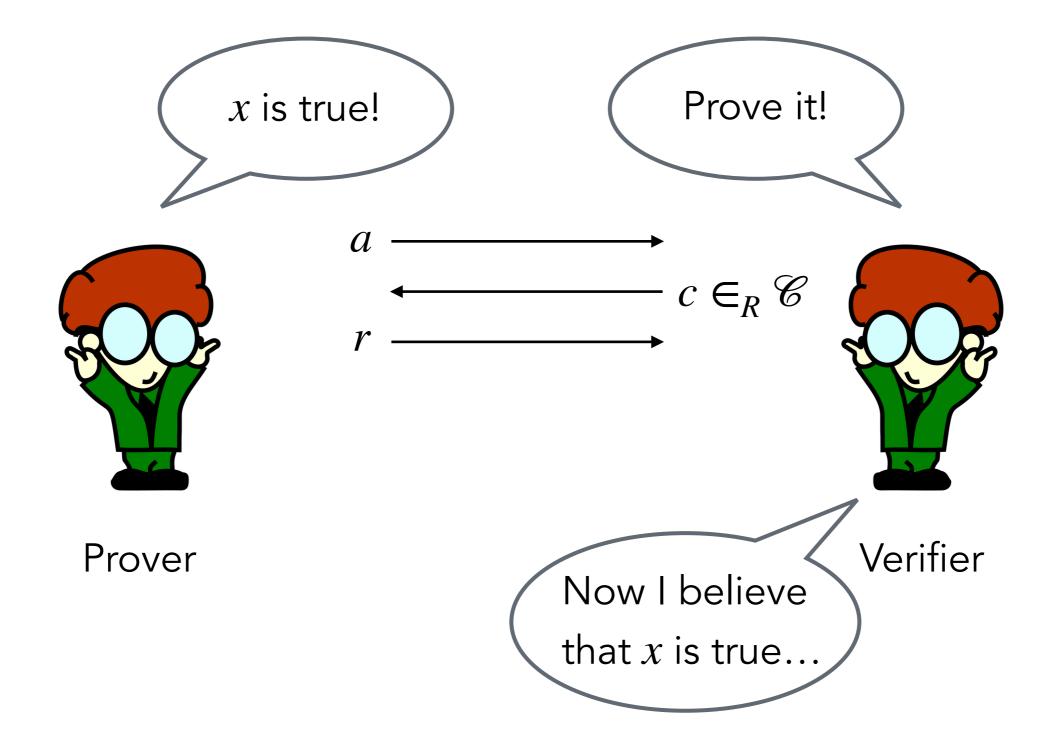
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Can we find a matching attack?

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#### Zero knowledge



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**Definition** (Honest-verifier zero knowledge, informal): A sigma protocol  $\Sigma$  is honest-verifier zero knowledge (HVZK) if there exists a simulator S such that for all true statements x,  $(a, c, r) \leftarrow S(x)$ is indistinguishable from a transcript from the protocol.

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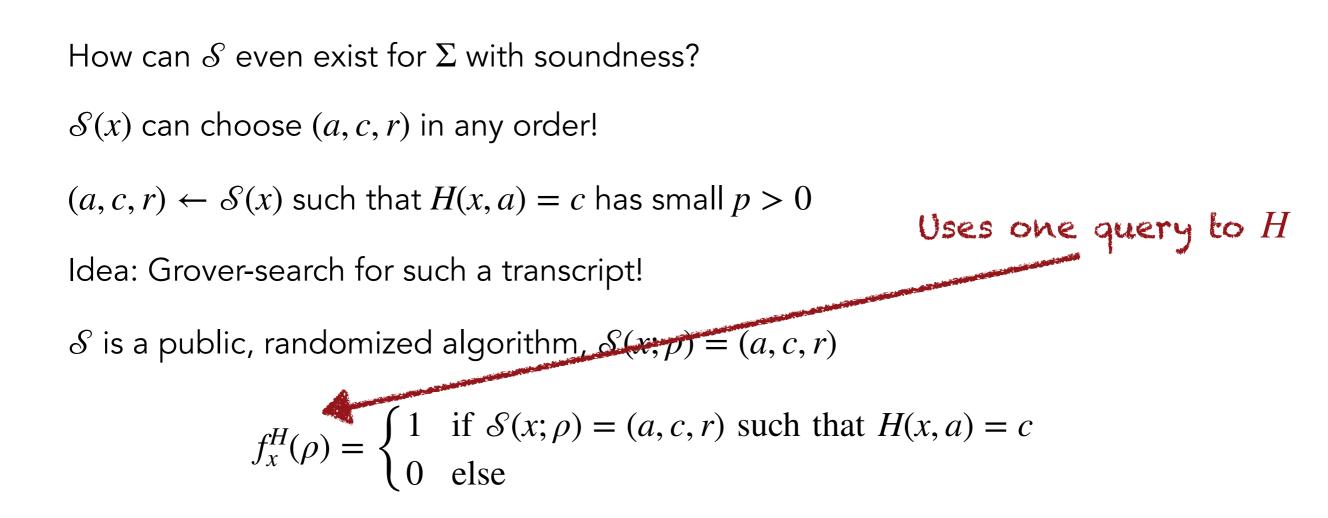
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$$f_x^H(\rho) = \begin{cases} 1 & \text{if } \mathcal{S}(x;\rho) = (a,c,r) \text{ such that } H(x,a) = c \\ 0 & \text{else} \end{cases}$$



How can  $\mathscr{S}$  even exist for  $\Sigma$  with soundness?  $\mathscr{S}(x)$  can choose (a, c, r) in any order!  $(a, c, r) \leftarrow \mathscr{S}(x)$  such that H(x, a) = c has small p > 0Uses one query to Hldea: Grover-search for such a transcript!  $\mathscr{S}$  is a public, randomized algorithm,  $\mathscr{S}(x;p) = (a, c, r)$  $f_x^H(\rho) = \begin{cases} 1 & \text{if } \mathscr{S}(x;\rho) = (a, c, r) \text{ such that } H(x, a) = c \\ 0 & \text{else} \end{cases}$ 

**Theorem** (informal; Don, Fehr, M '20):

Let  $\Sigma$  be a sigma protocol that is perfectly HVZK and has special soundness + some mild additional properties. Then there exists a quantum polynomial-time attacker making q queries to H that succeeds with probability  $\varepsilon_{FS(\Sigma)}(q) \ge q^2 \varepsilon_{\Sigma}$ .

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How relevant is the attack?

Sigma protocols for Fiat-Shamir signatures

- are HVZK
- Have special soundness or similar

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Better attacks possible, but likely using structure of H.

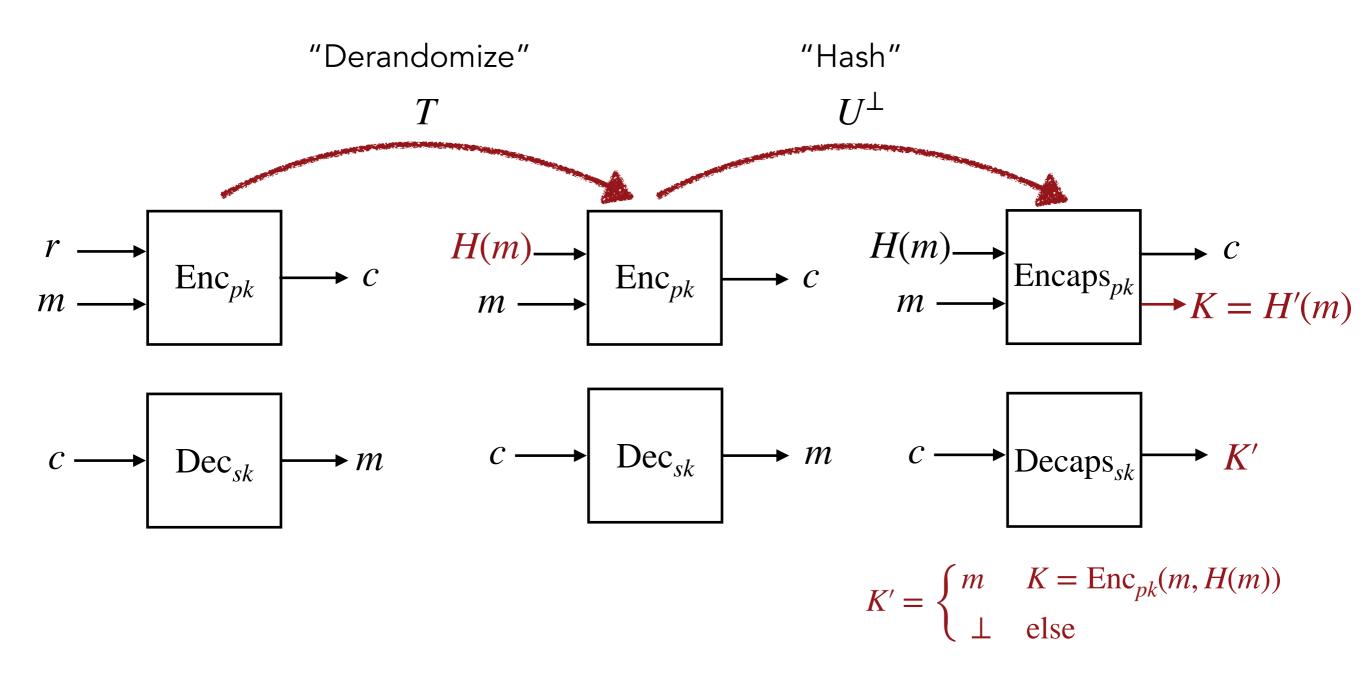
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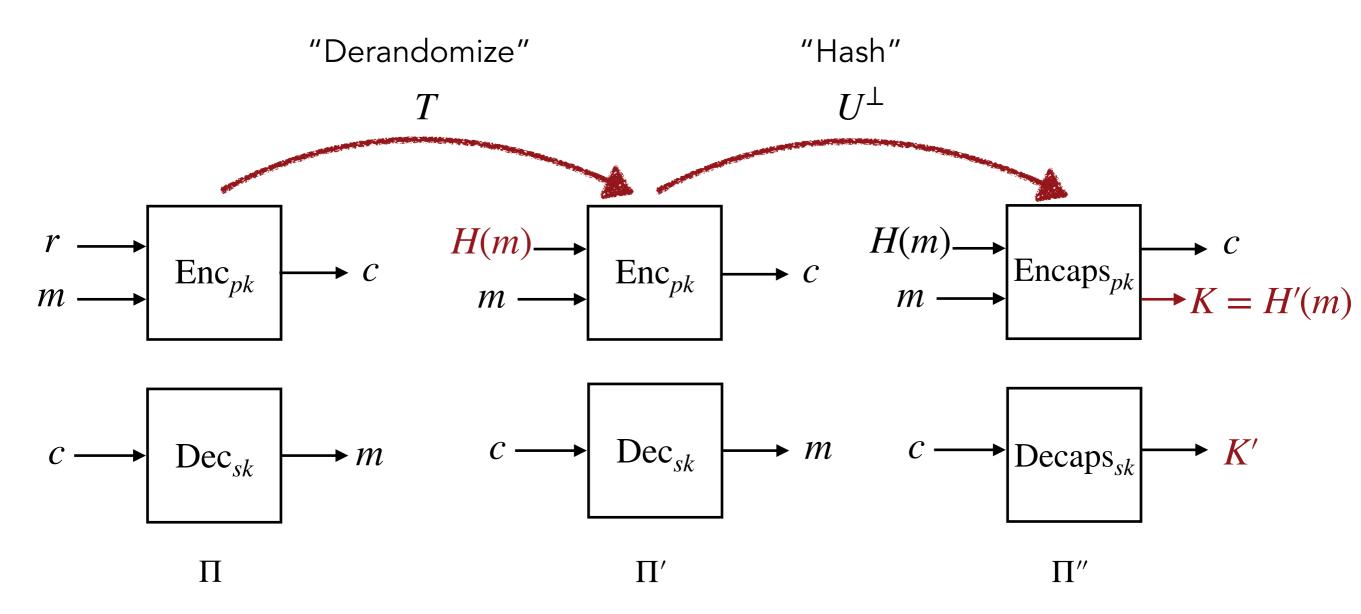
"Derandomize, then Hash"



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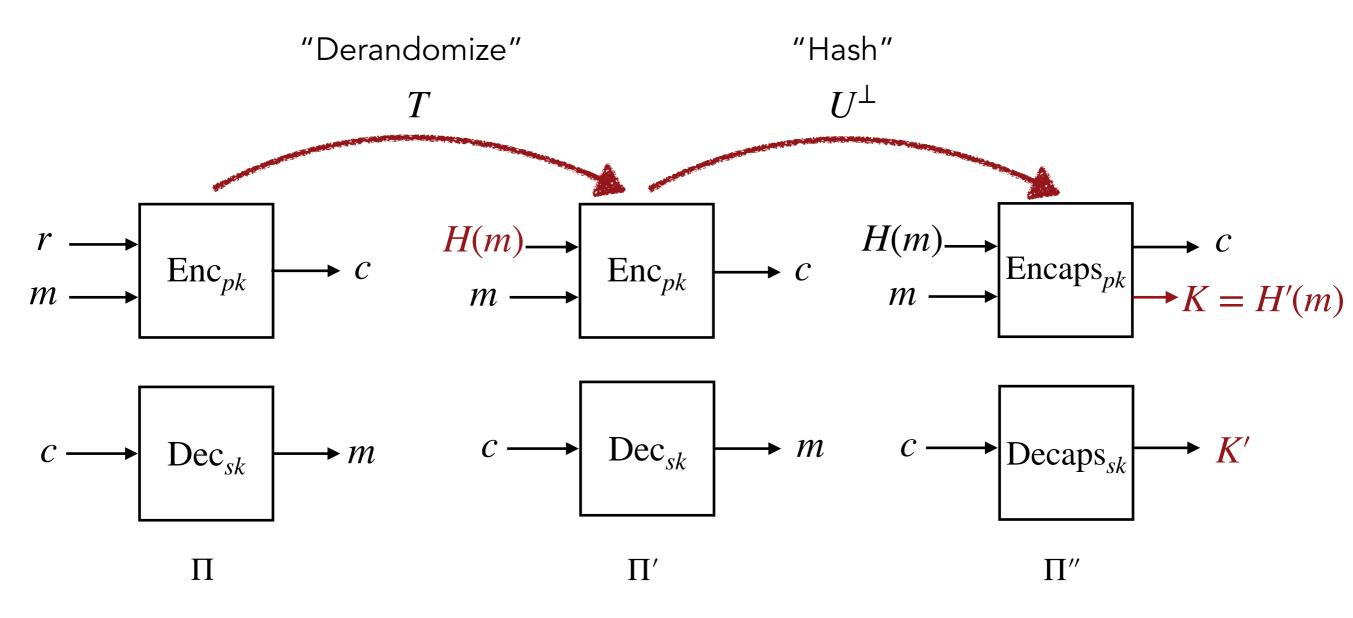
"Derandomize, then Hash"



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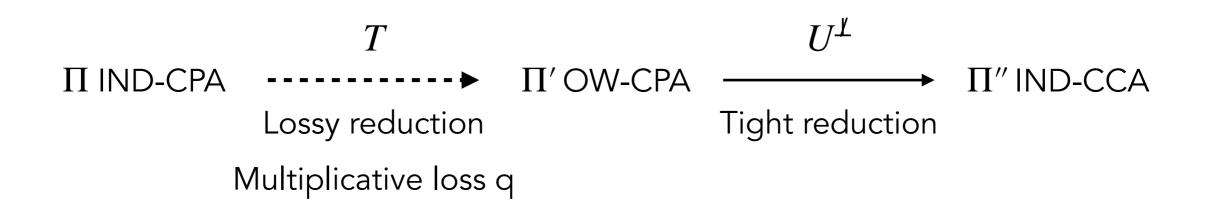
For proving post-quantum security, model H, H' as random oracles (QROM)

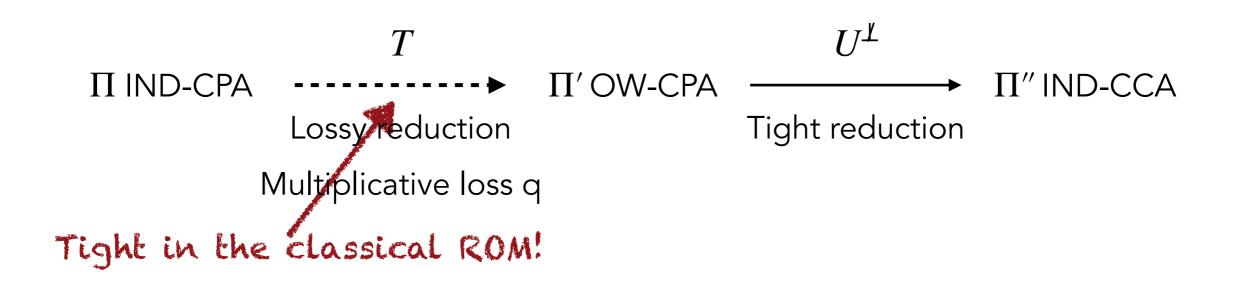
 $\Pi$  IND-CPA

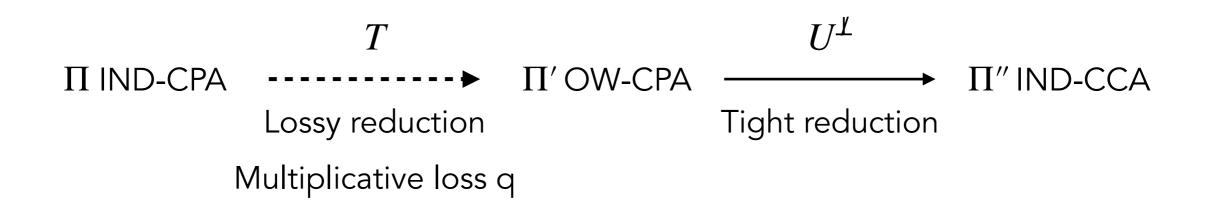
 $\Pi'\,\text{OW-CPA}$ 



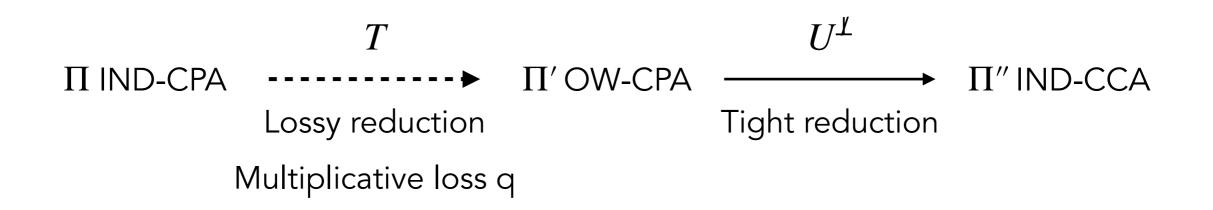






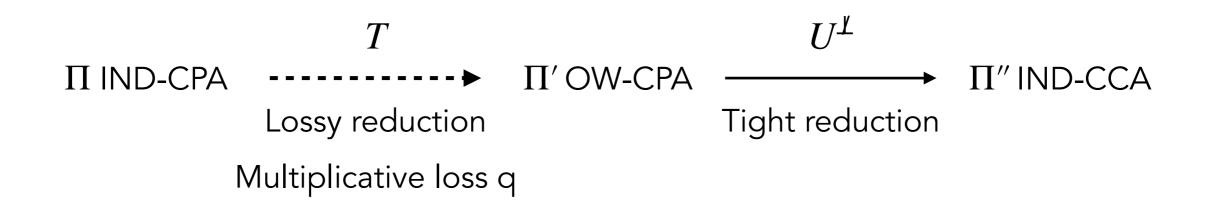


No attack known that exploits this gap



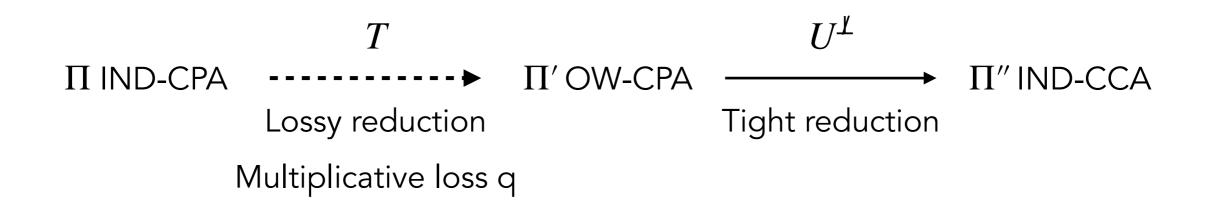
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Vanilla approach (Grover)?



No attack known that exploits this gap

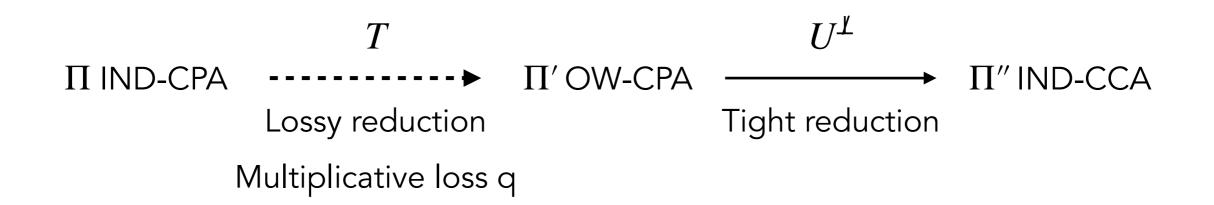
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Other algorithms?

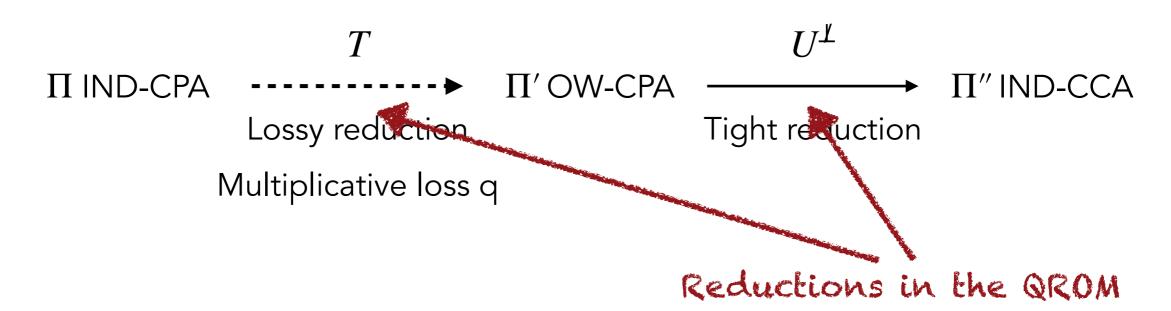


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(This is the question from Dan's email)

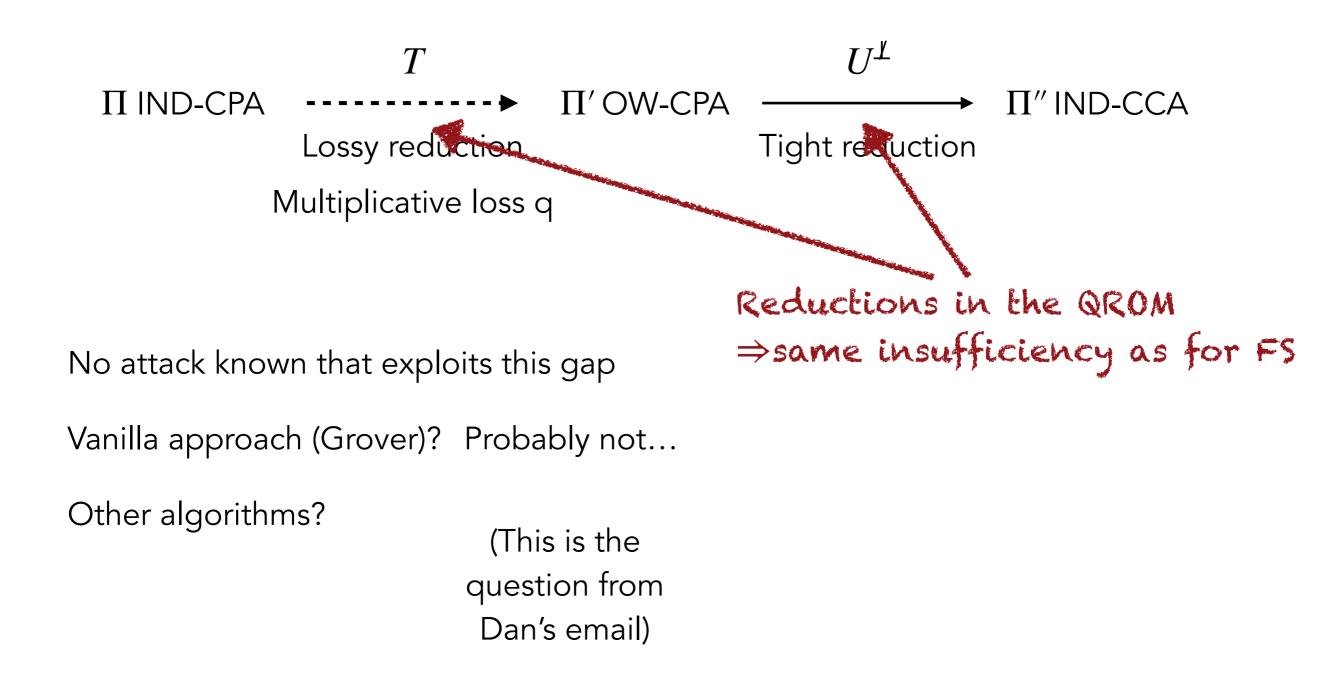


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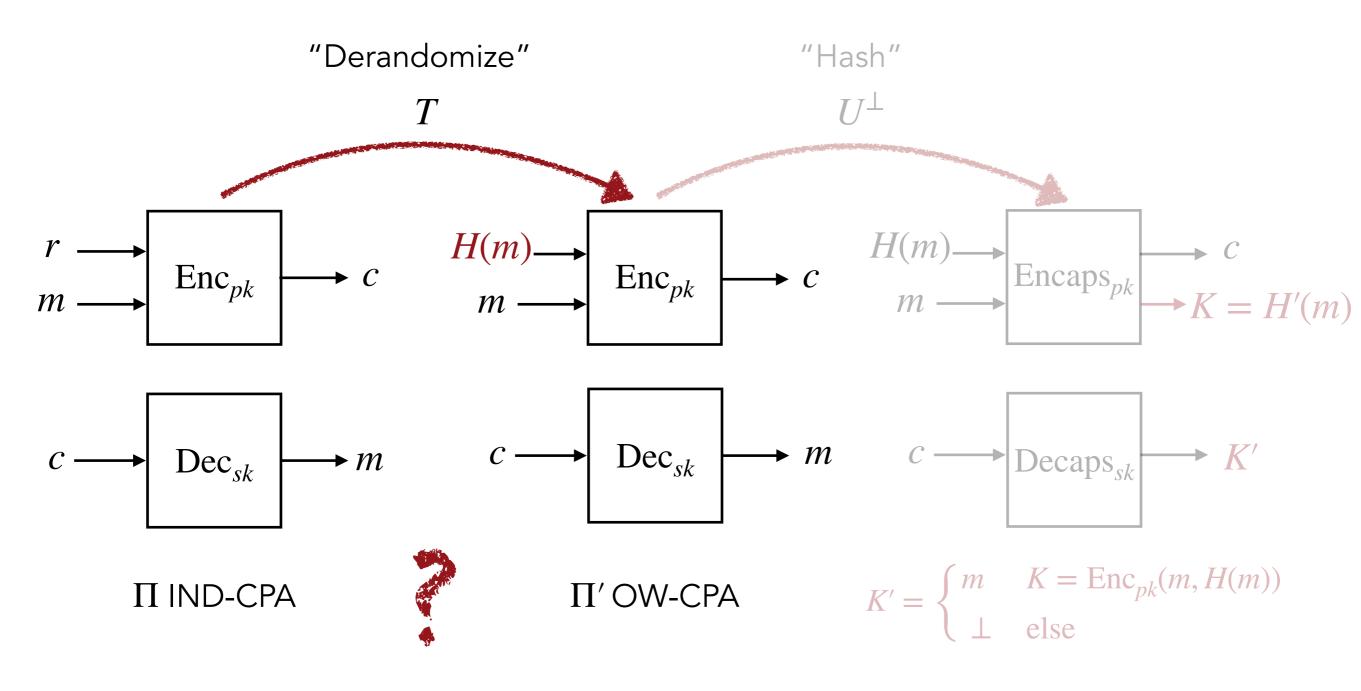
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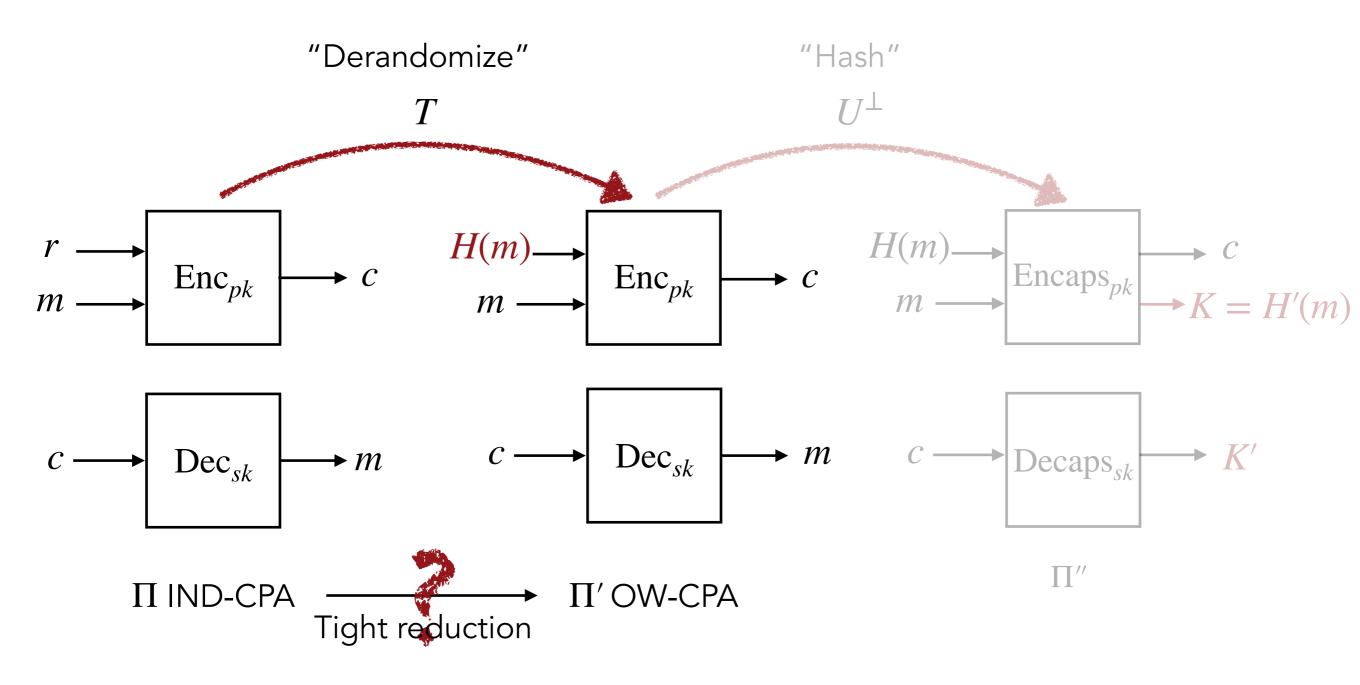
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#### Summary

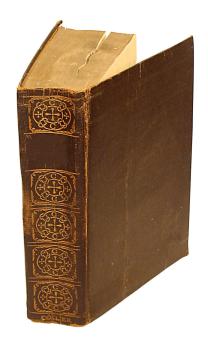
Hash functions are used everywhere.  $\Rightarrow$ We need to subject them to quantum cryptanalysis!

Attacks possible at different levels

Hash function application in schemes: some open questions regarding attacks

Polynomial improvements over trivial, but: important for parameter choice





# Thanks!



